Introduction to Flood Routing Methods

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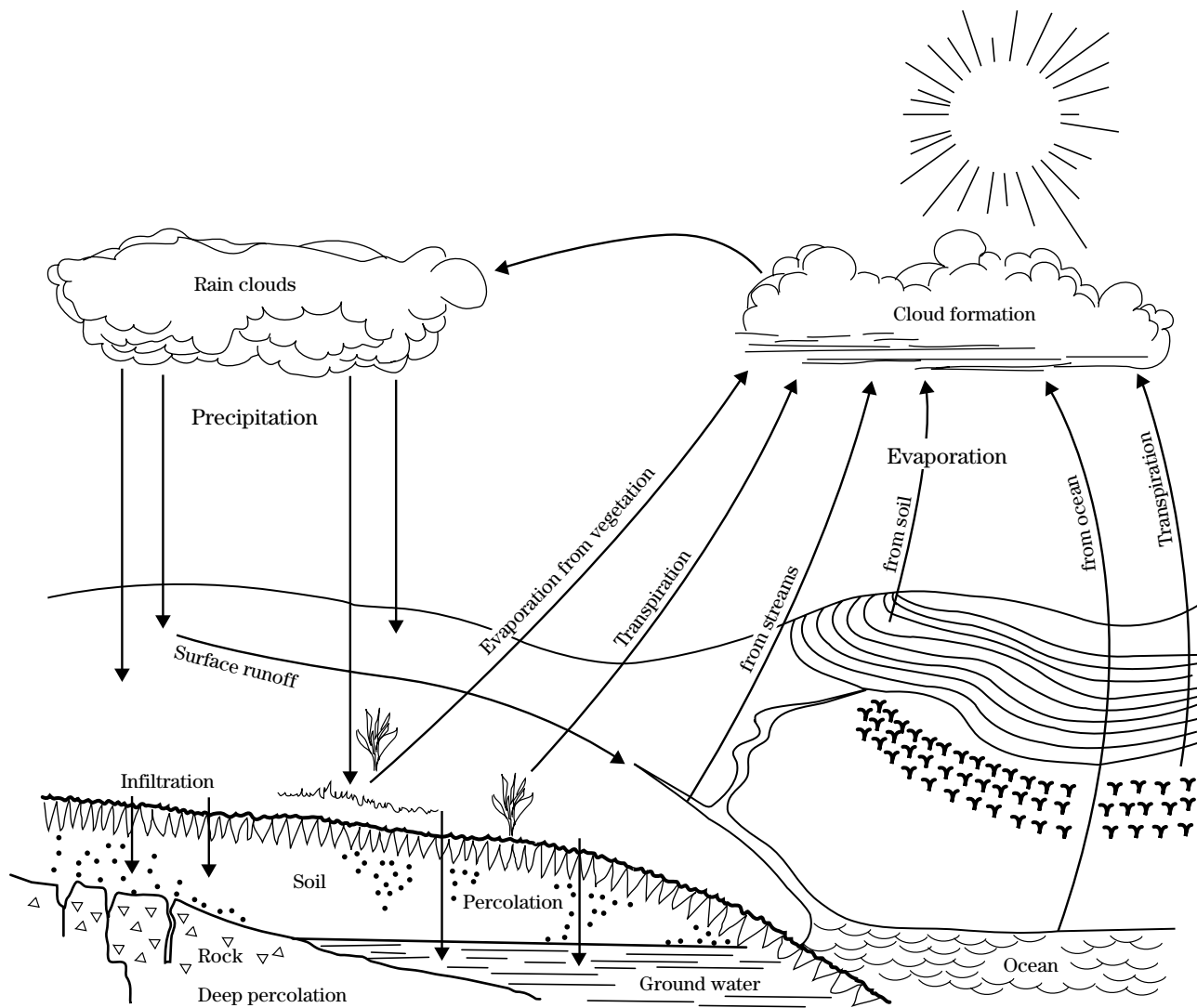
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630.1700 Introduction

Flood routing and hydrograph generation (see Part 630 Hydrology, chapter 16, 2007) form the key elements of a watershed hydrologic model. Stream channels, floodplains, and reservoirs can have a significant impact on the delivery of water to any location along a stream network. Flood routing impacts the magnitude of the peak discharge, the time of the peak discharge, depth and extent of flooding, and environmental factors such as stream bank erosion, floodplain scour, sediment transport, and deposition.

The examples in this chapter contain many tables generated by computer and spreadsheet. Some table values may differ from values calculated by other methods. Numerical accuracy is a function of the number of significant digits and the algorithms used in data processing, so some slight differences in numbers may be found if the examples are checked by other means.

630.1701 Basic concepts

(a) Definitions

Flood routing is variously defined as follows:

Routing, flood—The procedure that determines the timing and magnitude of a flood wave at a point on a stream from the known or assumed data at one or more points upstream (Chow 1964).

Routing, flow—A mathematical procedure that predicts the changing magnitude, speed, and shape of a flood wave as a function of time at one or more points along a watercourse (Maidment 1993).

Routing, stream channel—Mathematical relations that calculate outflow from a stream channel once inflow, lateral contributions, and channel characteristics are known (Ponce 1989).

These definitions relate to flood routing in streams and rivers. Definitions related to reservoirs and breaching dams are:

Routing, reservoir—This procedure derives the outflow hydrograph from a reservoir from the inflow hydrograph into the reservoir with consideration of elevation, storage, and discharge characteristics of the reservoir and spillways. The conservation of mass equation is solved with the assumption that outflow discharge and volume of storage are directly related.

Routing, breach—This procedure derives the hydrograph at downstream locations from the hydrograph generated by a breach or failure of a dam. This routing may include hydrographs resulting from precipitation in addition to the breach hydrograph.

Throughout this document, when a variable first appears in an equation, it is defined in the list of variables just below the equation. It is also listed in appendix D with its definition and a list of the equations in which it is found. After the first occurrence in an equation, the variables are not defined again with each usage; the reader should refer to appendix D.

(b) Hydrologic and hydraulic flood routing methods

Many textbooks provide detailed descriptions, derivations, and examples of various flood routing methods. Two general divisions or classifications of flood routing methods are hydrologic and hydraulic. Hydrologic methods are generally based on the solution of the conservation of mass equation and a relation of storage and discharge in a stream reach or reservoir. Hydraulic methods are based on solutions of the conservation of mass and the conservation of momentum equations. These methods have many formulations and solution techniques, many of which were developed before computers were available. Computers have saved much time and also allowed more robust solutions to the flood routing problem. It is still beneficial to understand how flood routing can be done manually so the engineer can analyze the impact of the various assumptions, limits of application, sensitivity of results, and adequacy of computer program results. Data requirements vary for each type of flood routing method. Choice of a method involves understanding the flow characteristics being modeled; available physical data such as flow data, stream and floodplain topography, roughness, etc.; and the purpose of the hydrologic analysis with associated risk.

Hydrologic flood routing methods are described in this chapter because of their inclusion in U.S. Department of Agriculture (USDA) Natural Resources Conservation Service (NRCS) computer programs. Hydraulic methods are available from several sources including the U.S. Army Corps of Engineers Hydrologic Engineering Centers River Analysis System (HEC–RAS 2010), U.S. Geological Survey (1981), and the National Weather Service (Fread and Lewis 1988).

Hydrologic methods have been used for many years and have proven their utility in analyzing flood conditions along streams and rivers. In general, hydrologic flood routing methods involve simplified numerical techniques, conservation of mass, and steady flow hydraulics. The hydraulic flood routing methods involve complex numerical solutions of partial differential equations and the theory of unsteady flow hydraulics. Hydrologic routing methods require a relation of discharge and storage, which can be derived from water surface profiles. Hydraulic methods do not require such a relation. Typical input data for hydraulic rout-

ing methods consists of topographic data such as channel geometry and reach lengths, roughness, downstream boundary conditions, and inflow hydrographs.

The movement of flood waves in streams is complex. Linsley, Kohler, and Paulhus (1982) describe two general classes of natural flood waves. The first includes waves in which the forces of momentum and acceleration control. These waves are fast rising and occur frequently on streams of steep gradient. The second class includes flood waves in which friction is the predominant force. Slow rising flood waves on streams of mild gradients are examples of this class.

The movement of a flood wave down a stream system may be described in terms of translation and storage effects. These represent characteristics of the two general classes of natural flood waves. Translation involves maintaining the same hydrograph shape as the flood wave moves downstream (fig. 17–1). The storage or attenuation effect involves use of valley storage to reduce the peak flow and change the shape of the hydrograph (fig. 17–2). These are not necessarily mutually exclusive conditions. Most flood waves exhibit both characteristics, though one may be more evident than the other based on the specific local situation. These two characteristics describe the upper and lower limiting conditions of the flood routing in a channel reach when considered individually. For example, the upper limit of flood routing is translation of the peak discharge with no attenuation as the flood wave moves downstream. If storage effects alone act on a flood wave, the result is maximum attenuation, so the peak discharge may not be further attenuated. This concept represents the lower limit to the flood routing of a channel reach.

These two characteristics of natural flood waves, translation and storage, are represented by the conservation of mass and conservation of momentum equations. The conservation of mass equation, sometimes referred to as the "continuity equation," is a mass balance and represents storage effects. It states that inflow minus outflow is equal to change in storage between two locations and between two moments in time. The conservation of momentum equation derives from the basic equation of physics, which states that force equals mass times acceleration, arranged in terms related to movement of water in a stream such as discharge, end-area, depth, top width, etc. This equation also applies between two locations and be-

Figure 17–1   Flood wave at two points along a channel exhibiting translation effects
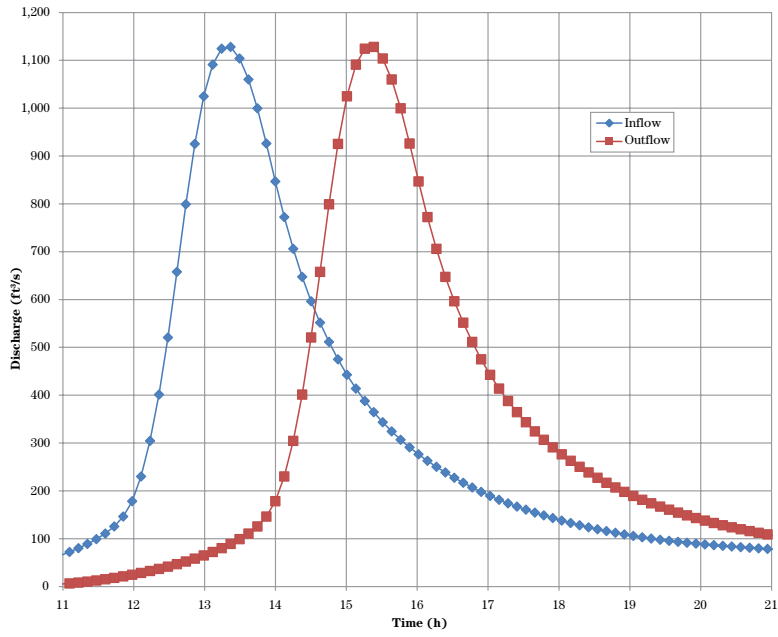
Figure 17–2   Flood wave at two points along a channel exhibiting storage effects
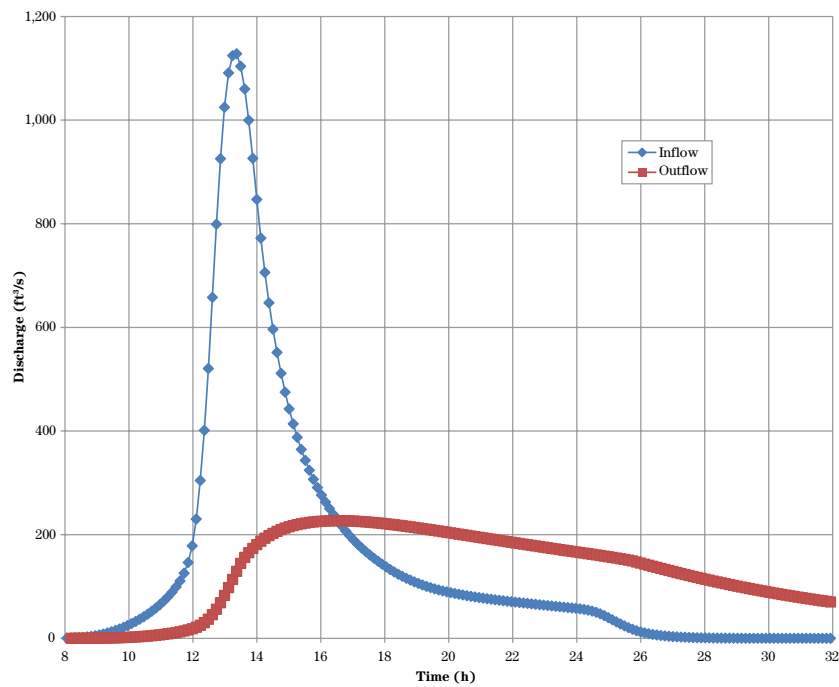
tween two moments in time. The solution of these two equations represents the hydraulic routing method.

Solving these equations for the extreme conditions of pure storage and pure translation provide the lower and upper limits of downstream peak discharge, respectively. Most flood routing situations are a combination of translation and storage effects. By solving the conservation of mass and conservation of momentum equations, the most physically based solution may be reached.

Hydrologic flood routing methods use the conservation of mass equation with simplifying assumptions to reduce the conservation of momentum equation to a relation of discharge to storage. The primary assumption is that acceleration terms are negligible with respect to other terms in the equation and are ignored. In many flood routing situations, this is a reasonable assumption. The acceleration terms are relatively small for slow-rising flood waves on moderate gradient streams. This is not a good assumption when the stream gradient is very small, which makes the acceleration terms more important even when they themselves are small, or for fast rising hydrographs such as those from a breaching dam, where the acceleration terms are relatively large.

For reservoirs of the general size considered by the NRCS, the assumption of a level pool is reasonable and the conservation of mass equation along with a relation of discharge and storage is solved. The description on reservoir routing describes the storage-indication routing method, which is appropriate for most NRCS applications.

For channel and valley flood routing applications, the Muskingum-Cunge method of flood routing is described. The Muskingum-Cunge method is classified as a hydrologic method because it requires a relation of discharge and storage. This relation is normally developed from steady flow water surface profiles, but may also be developed from a solution of Manning's equation for a cross section. This chapter has derivations, application strategies, and examples for applying the Muskingum-Cunge method.

(c)　NRCS computer programs

Routing of reservoirs and channels may be done with computer programs as part of a comprehensive hydrologic analysis of a watershed. Reservoir routing with the Storage-Indication method and channel routing with the Muskingum-Cunge method are integrated into the WinTR–20 (2010), SITES (2005), and WinTR–55 (2011) computer programs. Manual versions of both methods are described in this chapter with their data requirements, data preparation, assumptions, and limitations.

630.1702 Reservoir flood routing method

(a) Elevation-storage relations for reservoirs

One of the important data items needed to complete a reservoir routing is a relation of elevation or stage with volume of storage. If a reservoir is being constructed, elevation-storage information is needed from the lowest elevation at the centerline of dam up to an elevation slightly higher than the expected top of dam. For an existing reservoir, elevation-storage information is generally needed from the elevation of the permanent pool/principal spillway up to an elevation slightly higher than the top of dam. The presence of residential or rural development may influence the accuracy needed in development of the elevation-storage relation.

The relation is developed from a contour map or its equivalent of the reservoir area. A table is generally the easiest way to organize and document the calculations. Table 17–1 is a working table that shows data and computed results.

Once the contour map of a site is available,

Step 1: Select elevation increments that define the topography with reasonable accuracy, and tabulate them in column 1.

Step 2: Determine the reservoir surface area at each elevation. For this table the areas were determined in acres and tabulated in column 2.

Step 3: Compute average surface areas in column 3.

Step 4: Tabulate the increments of depth in column 4.

Step 5: Compute the increments of storage for column 5 by multiplying an average area in column 3 by its appropriate depth increment in column 4.

Step 6: Accumulate the storage increments of column 5 in column 6 for each elevation of column 1.

Table 17–1   Elevation, surface area, and storage at a reservoir site

Elevation (ft)	Surface area (acres)	Average surface area (acres)	Depth increment (ft)	Storage increment (acre-ft)	Accumulated storage (acre-ft)
(1)	(2)	(3)	(4)	(5)	(6)
570	0				0.00
		4.82	2	9.64	
572	9.64				9.64
		18.37	2	36.73	
574	27.09				46.37
		40.80	2	81.59	
576	54.5				127.96
		71.63	4	286.50	
580	88.75				414.46
		106.67	5	533.35	
585	124.59				947.81
		153.60	5	767.98	
590	182.6				1,715.79
		205.64	5	1,028.18	
595	228.67				2,743.96
		250.01	5	1,250.05	
600	271.35				3,994.01

The relation of data in columns 1 and 6 is plotted in figure 17–3 as an elevation-storage curve. English units are used in this example. If SI units are used, elevations would be in meters, areas in hectares, and storage values in hectare-meters.

(b) Elevation-discharge relations for reservoirs

The second important data item needed for reservoir routing is a relation of elevation or stage with discharge through the principal spillway, auxiliary spillway, and over the top of the dam if overtopping is anticipated during the routing. Select a set of elevations starting at the elevation of the lowest outlet and ending at the expected maximum routed water level, and calculate discharge at each elevation through the single or multiple spillways. The elevations selected should be the same as those for which the elevation-

storage relation has been developed in the previous step to eliminate interpolations needed to make elevations consistent in both tables. The end result will be a table of elevations with the appropriate storage and discharge.

The wide variety of spillway types, combinations of spillways, and the quantity of hydraulic references and software used to calculate these spillway ratings precludes their inclusion in this chapter. The procedures used to develop the discharges in this chapter are based on spillway capacity charts available in other references (such as Brater and King 1982).

An example of development of an elevation-discharge relation for a reservoir is shown in example 17–2. English units of feet and cubic feet per second are commonly used. If SI units are used, elevations would be in meters and discharge values in cubic meters per second.

Figure 17–3 Plot of elevation-storage data from table 17–1
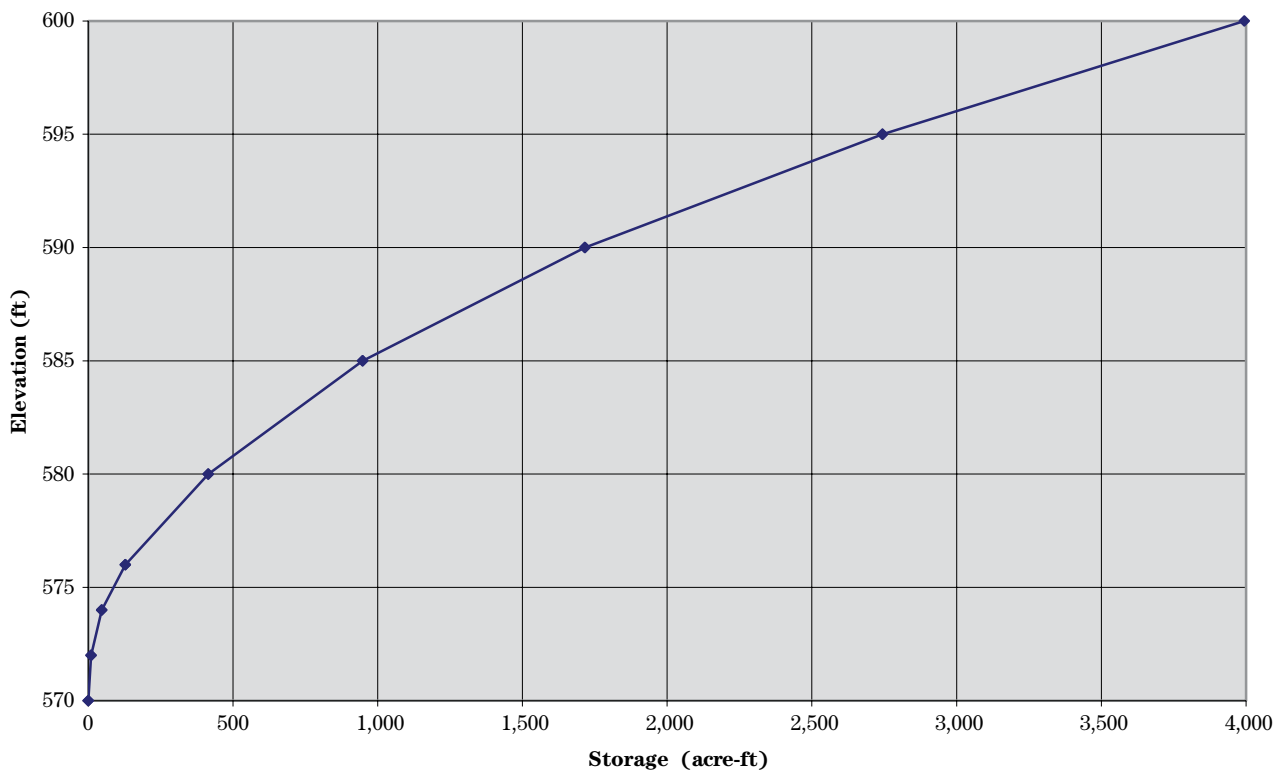
(c) Elevation-storage and elevation-discharge relations for channel reaches

Reservoir routing procedures may be used for channel reaches under certain circumstances. An example is a floodplain where a road or railroad fill backs up the water and forces flow to pass through a restrictive bridge or culvert. The decision to treat such a floodplain area as a reservoir requires judgment based on each case's physical characteristics:

- Is the floodplain relatively wide and the bridge or culvert restrictive?

- Does the history of flooding in the area indicate significant storage of water?

- Could the situation be treated as a reservoir without overtopping occurring?

- Is the slope of the stream and floodplain flat enough that the backwater would extend a significant distance upstream?

- Does enough storage in a combination of depth, width, and length exist to significantly impact the routing of the inflow hydrograph?

A rule of thumb is that if the volume of storage is more than 10 percent of the volume of the inflow hydrograph, the area should be treated as a reservoir.

The answers to many of these questions may be derived from water surface profiles and preliminary estimates of peak discharge and volume of runoff at the location. In a typical case, the 100-year flood may cause backwater with a reservoir-like situation and smaller floods will not. Depending on the purpose of the hydrologic analysis, the decision to treat the area as a reservoir is generally applied to all floods consistently.

Development of an elevation-discharge curve and water surface profiles for a stream reach is covered in NEH630.14. Typically, elevation-discharge curves are developed using a water surface profile computer program, such as HEC–RAS or solutions of Manning's equation. In this chapter, it is assumed that water surface profiles exist. Water surface profiles are generally computed for multiple discharges ranging from baseflow to the expected peak flow. In the example of

a road fill that restricts flow, the downstream end of the reach is at the road. The upstream end of the reach is where the water surface profile gradually changes from backwater with a nearly flat surface to normal flow where the slope of the water surface is about the same as the slope of the channel bottom. This exact location may be difficult to determine precisely because the position where the flow transition occurs usually depends on discharge. Typically, low discharges return to normal flow closer to the road than do high discharges. In making the selection of the upper limit of the reach, other factors may need consideration such as total length of reach, location of tributaries, and roads or other structures upstream requiring individual routings.

Elevation-discharge curves for a stream reach can be prepared using the procedure for reservoirs, but ordinarily a modified approach is used, and the storage-discharge curve is prepared directly. If the reach in question has several cross sections, they can be weighted according to the distances between cross sections and the total length of reach.

If the reach has three or more cross sections, a weighting method is needed. With only one or two cross sections the weighting is eliminated, but the reach storage is not defined as accurately. The weights are then used in a working table to develop an elevation-discharge curve. The calculations in the working table are typically based on the assumption that steady flow occurs in the reach at all depths of flow.

Development of the storage-discharge curve goes as follows:

- Select a series of discharges from zero to a discharge greater than any to be routed. Run water surface profiles with constant discharges, and display the cross-sectional end-area in the computer output.

- Record the cross-sectional end-area, or flow area, for the discharge for each of the cross sections.

- Compute weighted end-areas for each profile.

- Sum the weighted end-areas for each discharge to obtain an average end-area for each profile.

- Compute storage for each profile using this equation:

$$S = \text{reach length} \times \frac{A}{3,600}$$

where:
S = storage in ft³/s-h
reach length = ft
A = the average end-area

The unit conversion factor to convert seconds to hours is 3,600.

- Compute reach storage in acre-feet for each profile by multiplying the storage values by the reach length in feet and dividing by 43,560 square feet per acre. The units of cubic feet per second-hour are generally used for storage-indication routing for convenience.

A hypothetical storage-discharge curve is plotted in figure 17–4. Data in this plot can be used to prepare a working curve for routing. Example 17–1 is an example of computation of a discharge-storage relation for a channel. The complete storage-indication routing using a working curve is shown in example 17–1.

If steady flow water surface profiles are computed using the HEC–RAS computer program, accumulated storage (HEC–RAS output variable Volume) may be requested in an output table. The storage for each reach is tabulated in acre-feet or hectare-meters for each profile and represents storage from the lower end of the reach up to any particular cross section. The storage in the area of concern may be computed as the difference in accumulated storage from the upstream cross section minus the accumulated storage at the downstream cross section for each profile.

Figure 17–4   Plot of hypothetical discharge versus storage curve for a channel reach
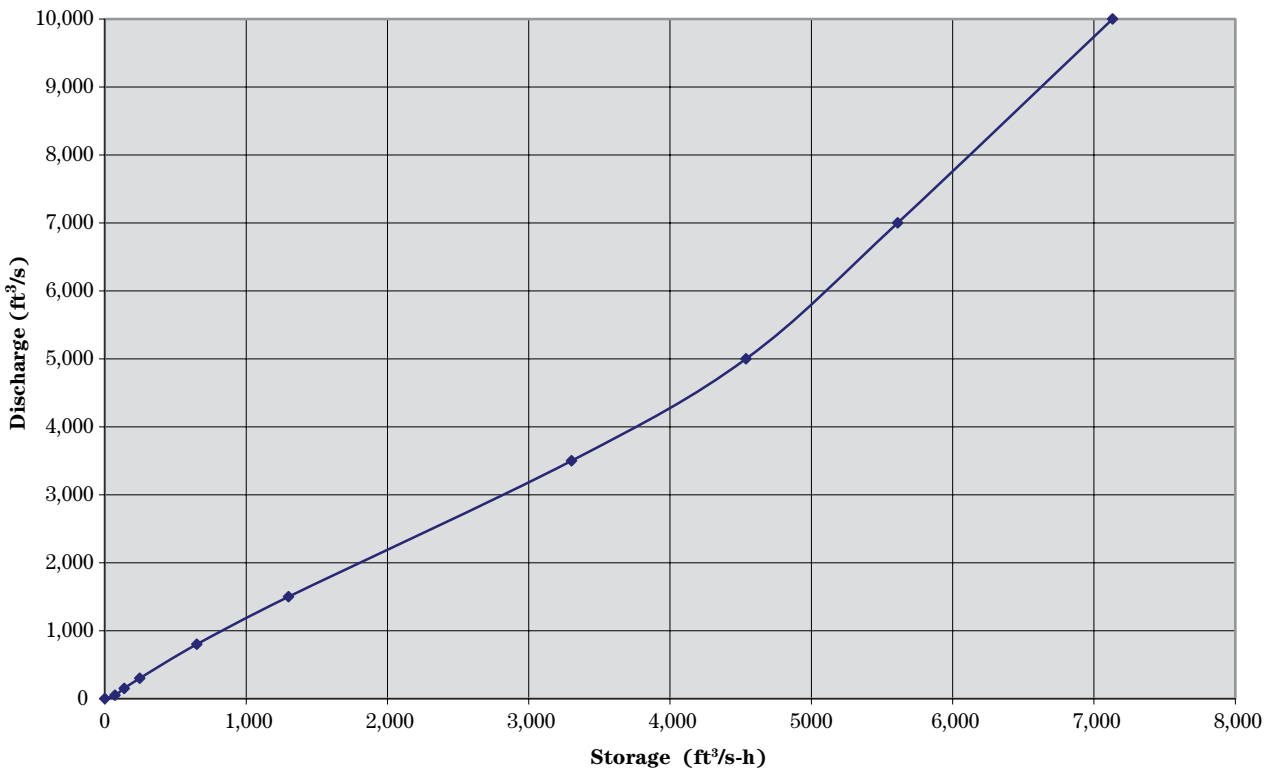
(d) Inflow discharge hydrograph to reservoirs or channel reaches

The third important data item needed to complete a reservoir routing is an inflow hydrograph representing discharge coming from the contributing watershed. This hydrograph may represent the hydrograph from any of various flood return periods, a hydrograph representing an actual flood event (NEH630.16; WinTR–20) or a design hydrograph such as a principal spillway hydrograph, auxiliary spillway design hydrograph, freeboard design hydrograph, or probable maximum flood hydrograph (NEH630.21; SITES).

Baseflow, if any, is generally added to the inflow hydrograph as a constant value and the reservoir routing started at an elevation where baseflow will pass through the principal spillway. This elevation may be derived from the elevation-discharge relation developed for the spillways. Alternative methods for handling of baseflow may be found in textbooks and other technical literature (Linsley, Kohler, and Paulhus 1982).

(e) Storage-indication routing method

Reservoir storage is closely related to the outflow rate. In reservoir routing methods, the storage-discharge relation is used for repeatedly solving the continuity equation, each solution being a step in delineating the outflow hydrograph. A reservoir routing method is suited for channel routings if the channel has certain reservoir-like characteristics. Suitable channels are those with wetlands or other flat areas in the routing reach with a constriction or similar control at the foot of the reach. Reservoir routing methods are also suitable for routing through any stream reach if the inflow hydrograph rises and falls so slowly that nearly steady flow occurs and makes storage in the reach closely related to the outflow rate. Examples 17–1 and 17–2 show the use of a reservoir routing method for both channels and reservoirs.

(f) The continuity equation

The continuity equation used in reservoir routing methods is that for the conservation of mass: for a given time interval, the volume of inflow minus the volume of outflow equals the change in volume of stor-

age. The principal assumption is that the water in the reservoir is level (level pool routing). The equation is often written in the simple form:

$$\Delta t \left(\overline{I} - \overline{O} \right) = \Delta S \qquad \text{(eq. 17–1)}$$

where:
Δt = time interval $t_2 - t_1$, h
   t_1 = time at the beginning of the time interval, h
   t_2 = time at the end of the time interval, h
$\overline{I}$ = average rate of inflow during the time interval, ft^3/s
$\overline{O}$ = average rate of outflow during the time interval, ft^3/s
ΔS = change in volume of storage during the time interval, ft^3/s-h

In most applications of the continuity equation, the flow and storage variables are expanded as follows:

$$\overline{I} = \frac{I_1 + I_2}{2}$$

$$\overline{O} = \frac{O_2 + O_1}{2}$$

$$\Delta S = S_2 - S_1$$

so that equation 17–1 becomes:

$$\frac{\Delta t}{2}\left(I_1 + I_2\right) - \frac{\Delta t}{2}\left(O_1 + O_2\right) = S_2 - S_1 \qquad \text{(eq. 17–2)}$$

where:
I_1 = inflow rate at t_1, ft^3/s
I_2 = inflow rate at t_2, ft^3/s
O_1 = outflow rate at t_1, ft^3/s
O_2 = outflow rate at t_2, ft^3/s
S_1 = storage volume at t_1, ft^3/s-h
S_2 = storage volume at t_2, ft^3/s-h
Δt = time interval, h

When routing with equation 17–1, the usual objective is to find the outflow. When equation 17–2 is used to solve for O_2, it must be rearranged in some more convenient working form. It is also necessary to use the relation of outflow to storage in making a solution. Most reservoir routing methods now in use differ only in their arrangement of the routing equation and in their form of storage-outflow relation.

It is necessary to use consistent units with the routing equation. Table 17–2 lists some of the commonly used sets of units.

The storage-indication method uses equation 17–2 in the form:

$$\frac{I_1 + I_2}{2} + \left(\frac{S_1}{\Delta t} + \frac{O_1}{2}\right) - O_1 = \frac{S_2}{\Delta t} + \frac{O_2}{2} \qquad \text{(eq. 17–3)}$$

A working curve (tabular and/or plotted) of O_2 against $(S_2 / \Delta t) + (O_2 / 2)$ is necessary for solving the equation.

The storage-indication method has the characteristic that the outflow begins at the same time as inflow begins so that the inflow at the upstream end of the reservoir or channel passes instantaneously through the reservoir or channel regardless of its size or length. This does not cause significant problems in channel routing if the ratio of T_t / T_p is about 0.5 or less, where T_p is the time of rise to the peak of the inflow hydrograph. If there is a long period of time where the hydrograph rises slowly then at a certain point rises rapidly to the peak, a practical way to define the value of T_p is to use a starting time where the discharge is 5 percent of the peak discharge. Subtract this time from the time to peak of the hydrograph to estimate T_p. T_t is the travel time defined as:

$$T_t = \frac{LA}{3600Q} = \frac{L}{3600V} \qquad \text{(eq. 17–4)}$$

where:
T_t = reach travel time in hours or the time it takes a selected steady flow discharge to pass through the reach
L = reach length, ft
A = end-area for discharge Q, ft^2
Q = flow rate or discharge, ft^3/s
V = Q/A = velocity of discharge Q, ft/s

In determining T_t, the discharge Q is usually the bankfull discharge under steady flow conditions. If this guideline for T_t / T_p is not met, it is recommended to split the reach into two or more routing distance steps and route each one sequentially. The concept of the routing distance step is that the reach length is divided into a number of equal lengths with the inflow hydrograph being routed through each length sequentially.

This guideline for T_t / T_p was developed by Victor Mockus. The basis for this guideline is described in an early version of chapter 17 (1962). His major concern was to not let reaches get too long. An assumption of the storage indication method is that the water surface is level (or nearly level). If the reach is very long, as the flood wave moves through the reach, it does not meet this assumption. Using the storage indication method in this situation will cause too much attenuation.

There is no rule for selecting the proper size of the routing time interval in using the storage-indication flood routing method for either reservoir or channel routing. Trial routings show that negative outflows will

Table 17–2   Units commonly used in routing equations

Units	Inflow rate	Outflow rate	Inflow volume	Outflow volume	Storage volume
English	ft³/s	ft³/s	ft³	ft³	ft³
English	in/h	in/h	in	in	in
English	ft³/s	ft³/s	ft³/s-h	ft³/s-h	ft³/s-h
English	ft³/s	ft³/s	ft³/s-d	ft³/s-d	ft³/s-d
English	acre-ft/d	acre-ft/d	acre-ft	acre-ft	acre-ft
English	in/d	in/d	in	in	in
Metric	m³/s	m³/s	m³	m³	m³
Metric	m³/s	m³/s	m³/s-h	m³/s-h	m³/s-h
Metric	m³/s	m³/s	m³/s-d	m³/s-d	m³/s-d

occur on the recession side of the outflow hydrograph whenever Δt is greater than $2 S_2 / O_2$ (or whenever $O_2 / 2$ is greater than $S_2 / \Delta t$). This also means that the rising portion of the outflow hydrograph is distorted. In practice, to avoid these possibilities, the working curve can be plotted as shown in figure 17–5; if any part of the working curve falls to the left of the line of equal values, the entire curve should be discarded and a new curve computed using a smaller value of Δt. For channel routing, the possibility of negative flows is usually excluded by taking Δt less than T_t, the travel time through the reach. In any application of the storage-indication method in the WinTR–20 computer program, if this condition occurs, the Δt is adjusted internally and the routing completed.

Example 17–1 Storage indication method applied to channel routing

Example 17–1 illustrates the use of the storage-indication method applied to channel routing. The inflow hydrograph of figure 17–6 is routed using a storage-discharge relation developed for a stream reach.

The steps to calculate the routing are:

Step 1: Prepare the storage-discharge relation for the reach.

The reach in question has several cross sections which may be weighted as in table 17–3. To determine the weights for each cross section, use the distances between them compared to the total

Figure 17–5 Plot of working curve for example 17–1
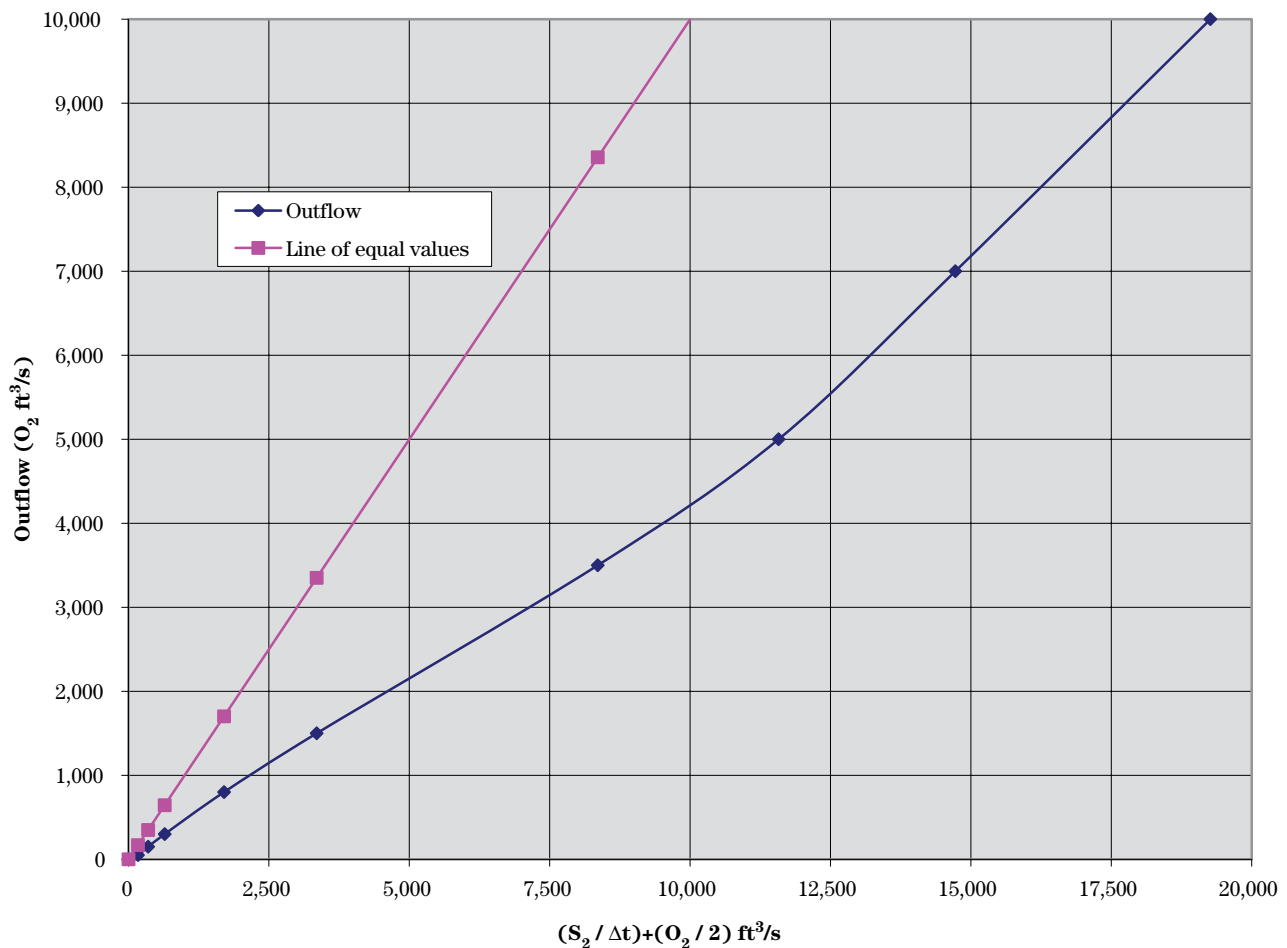
Figure 17–6 Inflow hydrograph for example 17–1
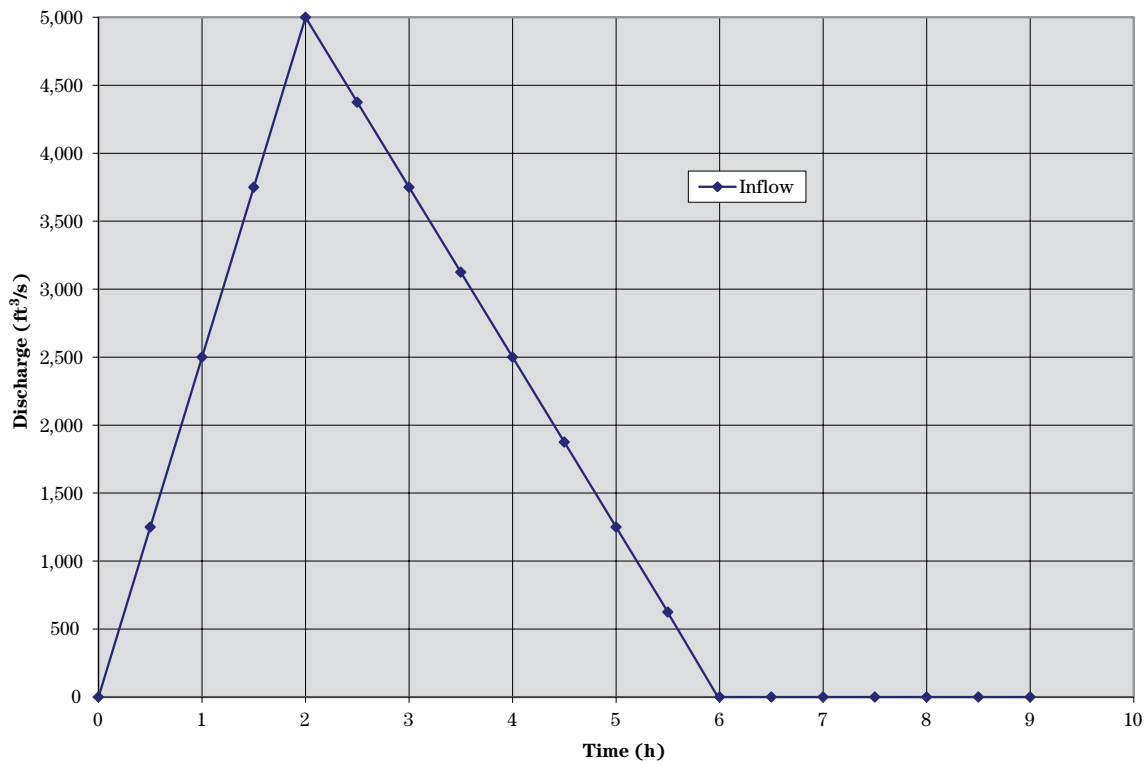
Table 17–3 Cross section separation distances and weights

From cross section	To cross section	Distance (ft)	Weight
1	2	1,000	0.10
2	3	6,000	0.60
3	4	3,000	0.30
		Sum=10,000	

length of reach. In table 17–3, the weight for sub-reach 1 to 2 is 1,000/10,000 = 0.10, and so on.

The reach used in tables 17–3 and 17–4 has four cross sections so a weighting method is needed. The weights are then used in table 17–4, a working table for developing an elevation-discharge curve. The calculations in table 17–4 are based on the assumption that steady flow occurs in the reach at all depths of flow.

Consider a case where the bankfull discharge under steady flow conditions is 800 cubic feet per second. The development of the storage-discharge curve is:

- Select a series of discharges from zero to a discharge greater than any to be routed; and tabulate them in column (1) of table 17–4.

- Enter the cross section end-area or flow area for the discharge for each of the four cross sections in columns (2), (3), (4), and (5). These values can be found by running water surface profiles with constant discharges and displaying the cross section end-area in the computer output.

- Compute weighted end-areas for columns 6, 7, and 8. For example, at a discharge of 3,500 cubic feet per section, cross section 1 has an end-area of 2,500 square feet, cross section 2 has 640 square feet, and the weighted end-area is 0.10 (2,500 + 640) divided by 2 is 157 square feet.

- Sum the weighted end-areas for columns 6, 7, and 8 for each discharge, tabulating the sums in column 9.

- Compute storage in column 10 by the equation:

$$S = 10,000 \times \frac{A}{3,600}$$

where:
S   = storage, ft³/s-h
10,000 = total reach length, ft
A   = sum of the weighted end-areas from column 9
3,600   = unit conversion factor

- Column 11 is computed by multiplying the values in column 9 by 10,000 feet and dividing by 43,560 square feet per acre. The units of cubic feet per second-hour are used for storage for convenience in the routing.

Table 17–4   Working table for development of elevation versus storage for a channel reach

Outflow	Cross section end-areas by cross section				Weighted end-area by cross section pairs			Sum of end-areas	Storage	Storage
	1	2	3	4	1–2	2–3	3–4			
(ft³/s)	(ft²)	(ft²)	(ft²)	(ft²)	(ft²)	(ft²)	(ft²)	(ft²)	(ft³/s-h)	(ac-ft)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	0	0	0	0	0.0	0.0	0.0	0.0	0.0	0.0
50	40	27	21	33	3.4	14.4	8.1	25.9	71.8	5.9
150	90	44	44	64	6.7	26.4	16.2	49.3	136.9	11.3
300	150	83	83	100	11.7	49.8	27.5	88.9	246.9	20.4
800	470	180	220	325	32.5	120.0	81.8	234.3	650.7	53.8
1,500	950	310	460	700	63.0	231.0	174.0	468.0	1,300.0	107.4
3,500	2,500	640	1,200	2,000	157.0	552.0	480.0	1,189.0	3,302.8	273.0
5,000	3,250	860	1,700	2,700	205.5	768.0	660.0	1,633.5	4,537.5	375.0
7,000	4,400	1,050	2,050	3,400	272.5	930.0	817.5	2,020.0	5,611.1	463.7
10,000	5,800	1,300	2,550	4,500	355.0	1,155.0	1,057.5	2,567.5	7,131.9	589.4

- The storage discharge curve in figure 17–7 is a plot of the data in columns 1 and 10. Data in those columns can be used to prepare the working curve for routing. For the storage indication method, the working curve is prepared as shown in this routing example.

Step 2:   Determine the reach travel time. Table 17–4 and the accompanying text supply the following data: L is 10,000 feet, and for a bankfull discharge of 800 cubic feet per second, the end-area is 234 square feet. From equation 17–4:

$$T_t = \frac{10,000(234)}{3600(800)} = 0.813 \text{ h}$$

From figure 17–6, the time to peak of the inflow hydrograph is 2.0 hours. The ratio of T_t / T_p is 0.4. Since this is less than the 0.5 guideline, the routing should be reasonable.

Step 3:   Select the routing time interval. The routing time interval for this example will be 0.5 hour, which is less than the travel time of step 2 and also is a convenient size for routing the given inflow hydrograph. The routing increment should be less than or equal to the calculated travel time in step 2.

Step 4:   Table 17–5 shows the working curve. Use the storage-discharge relation of step 1, which is given in columns 1 and 10 of table 17–4. These two columns are reproduced as columns 1 and 3 of table 17–5, the working table. Columns 2, 4, and 5 of the table are self-explanatory. The working curve is plotted using columns 1 and 5. The finished curve is plotted in figure 17–5.

Step 5:   Prepare the operations table, table 17–6, with suitable headings.

Step 6:   Enter times and inflows in the operations table. Accumulated time steps of the routing interval are shown in column 1 of table 17–6. Discharge values read from figure 17–6 at each time step on the inflow hydrograph are entered in column 2. Compute average inflow (column 3) for each time interval.

Step 7:   Do the routing by applying equation 17–3. The detailed procedure is shown in table 17–6. The first three time steps are explained as follows.

For the starting time of 0.0, average inflow, storage, and outflow are 0.0.

At time 0.5 hour, take average inflow (col. 3) of 625 and add $(S_2 / \Delta t) + (O_2 / 2)$ at time 0.0 (col 4) and subtract outflow at time 0.0 (col 5) to get 625. Interpolating from table 17–5, at a value of $(S_2 / \Delta t) + (O_2 / 2)$ of 625, outflow is 290.

At time 1.0 hour, take average inflow (col 3) of 1,875, and add $(S_2 / \Delta t) + (O_2 / 2)$ at time 0.5 (col 4) of 625, and subtract outflow at time 0.5 (col 5) of 290 to get 2,210. Interpolating from table 17–5, at a value of $(S_2 / \Delta t) + (O_2 / 2)$ of 2,210, outflow is 1,016.

The summary of routing results are shown in columns 2 and 5 of table 17–7. The inflow (column 2) and outflow (column 5) from table 17–7 are plotted in figure 17–8. Table 17–7 shows the complete routing results.

Figure 17–7   Plot of discharge versus storage for the channel reach in table 17–4
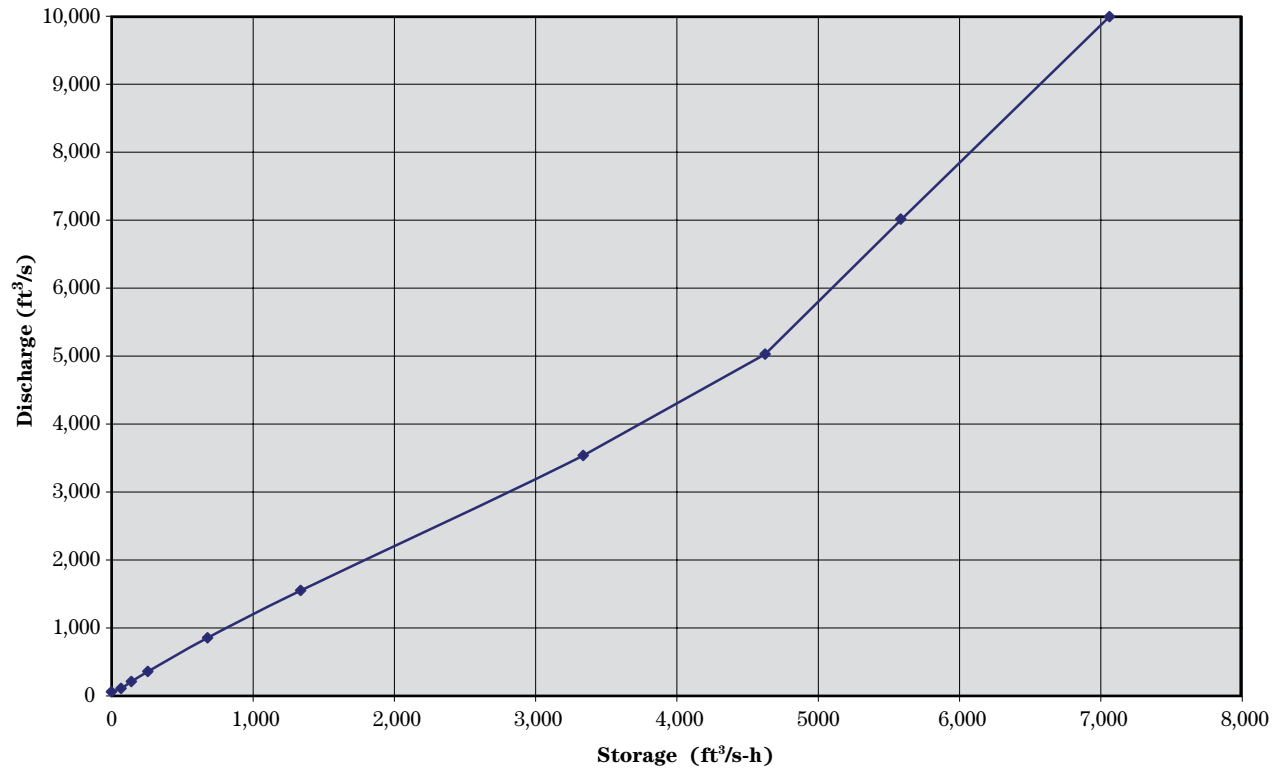
Table 17–5   Working curve data for example 17–1

Outflow			Storage	
O_2 (ft³/s) (1)	$O_2 / 2$ (ft³/s) (2)	S_2 (ft³/s-h) (3)	$S_2 / \Delta t$ (ft³/s) (4)	$S_2 / \Delta t + O_2 / 2$ (ft³/s) (5)
0	0	0.0	0.0	0.0
50	25	71.8	143.6	168.6
150	75	136.9	273.9	348.9
300	150	246.9	493.9	643.9
800	400	650.7	1,301.4	1,701.4
1,500	750	1,300.0	2,600.0	3,350.0
3,500	1,750	3,302.8	6,605.6	8,355.6
5,000	2,500	4,537.5	9,075.0	11,575.0
7,000	3,500	5,611.1	11,222.2	14,722.2
10,000	5,000	7,131.9	14,263.9	19,263.9

Figure 17–8   Inflow and outflow hydrographs for example 17–1
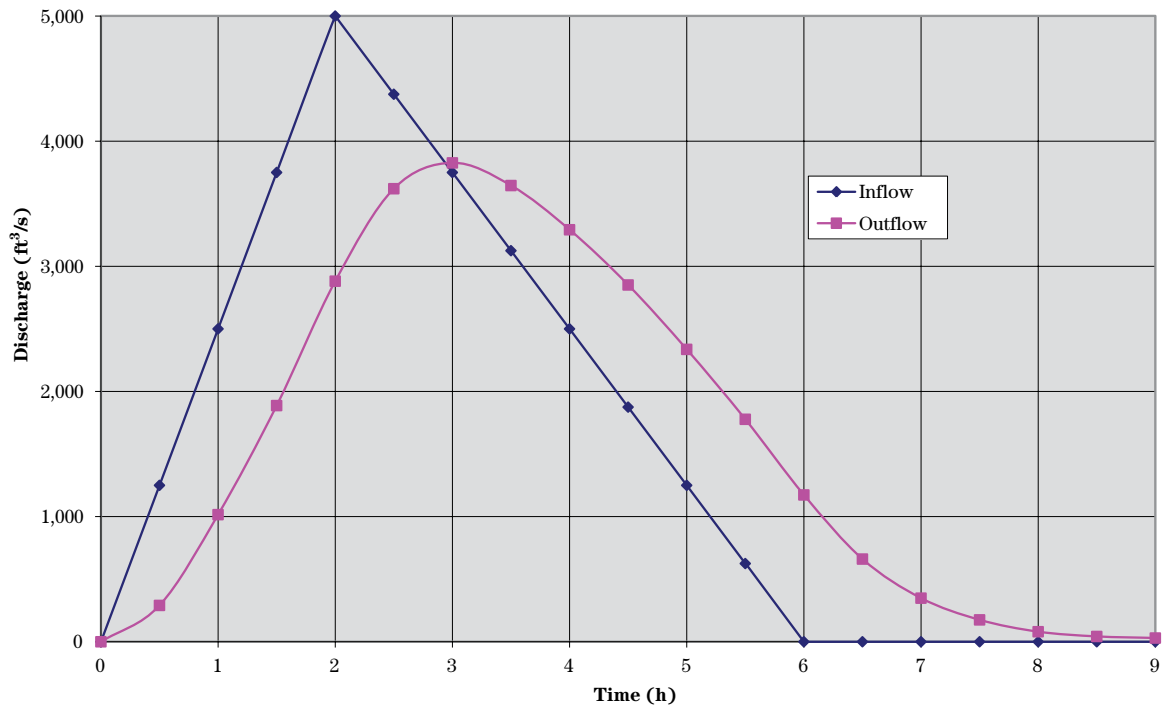
Table 17–6 Detailed operations table routing calculations for example 17–1

Time (h)	Inflow (ft³s)	Average inflow (ft³s)	$S_2/\Delta t + O_2/2$ (ft³s)	Outflow (ft³s)	Remarks
(1)	(2)	(3)	(4)	(5)	(6)
0.0	0	0	0	0	Given
0.5	1,250	625			Given
			625		$625 + 0 - 0 = 625$
				290	Interpolated from table 17–5
1.0	2,500	1,875			Given
			2,210		$1,875 + 625 - 290 = 2,210$
				1,016	Interpolated from table 17–5
1.5	3,750	3,125			Given
			4,319		$3,125 + 2,210 - 1,016 = 4,319$
				1,887	Interpolated from table 17–5
2.0	5,000	4,375			Given
			6,807		$4,375 + 4,319 - 1,887 = 6,807$
				2,881	Interpolated from table 17–5
2.5	4,375	4,688			Given
			8,614		$4,688 + 6,807 - 2,881 = 8,614$
				3,620	Interpolated from table 17–5
3.0	3,750	4,063			Given
			9,056		$4,063 + 8,614 - 3,620 = 9,056$
				3,827	Interpolated from table 17–5
3.5	3,125	3,438			Given
			8,668		$3,438 + 9,056 - 3,827 = 8,668$
				3,645	Interpolated from table 17–5
etc.	etc.	etc.	etc.	etc.	

Table 17–7 Summary operations table for example 17–1

Time (hours)	Inflow (ft³/s)	Avg. Inflow (ft³/s)	$S_2/\Delta t + O_2/2$ (ft³/s)	Outflow O (ft³/s)	Time (hours)	Inflow (ft³/s)	Avg. Inflow (ft³/s)	$S_2/\Delta t + O_2/2$ (ft³/s)	Outflow O (ft³/s)
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
0.0	0	0	0	0	5.0	1,250	1,563	5,443	2,336
0.5	1,250	625	625	290	5.5	625	938	4,045	1,778
1.0	2,500	1,875	2,210	1,016	6.0	0	313	2,580	1,173
1.5	3,750	3,125	4,319	1,887	6.5	0	0	1,407	661
2.0	5,000	4,375	6,807	2,881	7.0	0	0	746	348
2.5	4,375	4,688	8,614	3,620	7.5	0	0	398	175
3.0	3,750	4,063	9,056	3,827	8.0	0	0	223	80
3.5	3,125	3,438	8,668	3,645	8.5	0	0	143	42
4.0	2,500	2,813	7,835	3,292	9.0	0	0	100	30
4.5	1,875	2,188	6,731	2,851	etc				

In example 17–2, the solution of the storage-indication equation (eq. 17–3) for reservoir routing is identical with that for channel routing. The data required for the channel reach and reservoir are the same (discharge-storage curve and inflow hydrograph). The preparation of discharge-storage curves for a reservoir is somewhat different in that reservoir routing does not require a travel time.

Example 17–2   Use the storage-indication method to determine the minimum required storage for a flood water retarding structure

The inflow hydrograph to be routed is a principal spillway hydrograph (PSH) developed with procedures outlined in NEH 630.21. The PSH is given in columns of 1 and 2 of table 17–8.

Step 1:   A stage versus storage relation is required to flood route a reservoir. The relation is developed from a contour map or its equivalent of the reservoir area. Table 17–9 is a working table that shows data and computed results to be used in this example.

From the contour map of the site:

- Elevation increments that define the topography with reasonable accuracy were selected and tabulated in column 1.

- The reservoir surface area at each elevation was determined. For this table, the areas (in units of acres) are tabulated in column 2.

- Average surface areas were computed in column 3.

- The increments of depth were tabulated in column 4.

- The average area in column 3 was multiplied by its appropriate depth increment in column 4 to compute the increments of storage in column 5.

- The storage increments of column 5 were accumulated in column 6 for each elevation of column 1.

The relation of data in columns 1 and 6 of table 17–9 is plotted in figure 17–9 as an elevation-stor-

Table 17–8   Principal spillway hydrograph for example 17–2

Time (days) (1)	Inflow (ft³/s) (2)	Time (days) (1)	Inflow (ft³/s) (2)
0.0	0	5.3	393
0.5	70	5.4	312
1.0	79	5.5	267
1.5	84	5.6	217
2.0	88	5.7	200
2.5	99	5.8	184
3.0	110	5.9	174
3.5	128	6.0	164
4.0	156	6.5	138
4.5	245	7.0	118
4.6	269	7.5	106
4.7	308	8.0	94
4.8	380	8.5	88
4.9	522	9.0	82
5.0	2,002	9.5	75
5.1	1,049	10.0	67
5.2	577	etc	

Table 17–9   Elevation, surface area, and storage at a reservoir site

Elevation (ft) (1)	Surface area (acre) (2)	Average surface area (acre) (3)	Depth incre-ment (ft) (4)	Storage increment (acre-ft) (5)	Accu-mulated storage (acre-ft) (6)
570	0				0.00
		4.82	2	9.64	
572	9.64				9.64
		18.37	2	36.73	
574	27.09				46.37
		40.80	2	81.59	
576	54.5				127.96
		71.63	4	286.50	
580	88.75				414.46
		106.67	5	533.35	
585	124.59				947.81
		153.60	5	767.98	
590	182.6				1,715.79
		205.64	5	1,028.18	
595	228.67				2,743.96
		250.01	5	1,250.05	
600	271.35				3,994.01

age curve. English units are used in this example. If SI units are used, elevations would be in meters, areas in hectares, and storage values in hectare-meters.

The elevation-discharge relation for a reservoir is also required to flood route a reservoir. Data used in this example are shown in table 17–10 and plotted in figure 17–11. Column 1 lists elevations. Columns 2 and 3 show the discharge for the first stage weir and orifice in cubic feet per second. Columns 4 and 5 show the discharge for the second stage weir and orifice in cubic feet per second. Column 6 gives the total discharge for the principal spillway system in cubic feet per second. The procedure used to develop the discharges in the table is not shown here but is based on spillway capacity charts available in other references (Brater and King 1982).

Table 17–10 illustrates a useful way of keeping the work in order. By tabulating the data for different types of flow in separate columns and keeping the two stages separate, the total discharges are more easily summed. The elevations in table 17–9 were selected to adequately define the relation of stage versus discharge of the spillway. Note that the totals in column 6 are not merely sums of all the values in a row; the operation of the spillway must be understood when selecting the discharges to be included in the sum. At elevation 585, control shifts from weir to orifice for the first stage, so that the total discharge in column 6 equals 114.6 cubic feet per second. Similarly, at elevation 588.5, control shifts from weir to orifice for the second stage, and the total discharge in column 6 equals 353.0 cubic feet per second. The total discharges for the elevations between 585 and 588.5 are equal to the sum of columns 3 and 4. To combine the

Figure 17–9   Plot of elevation-storage data from table 17–9
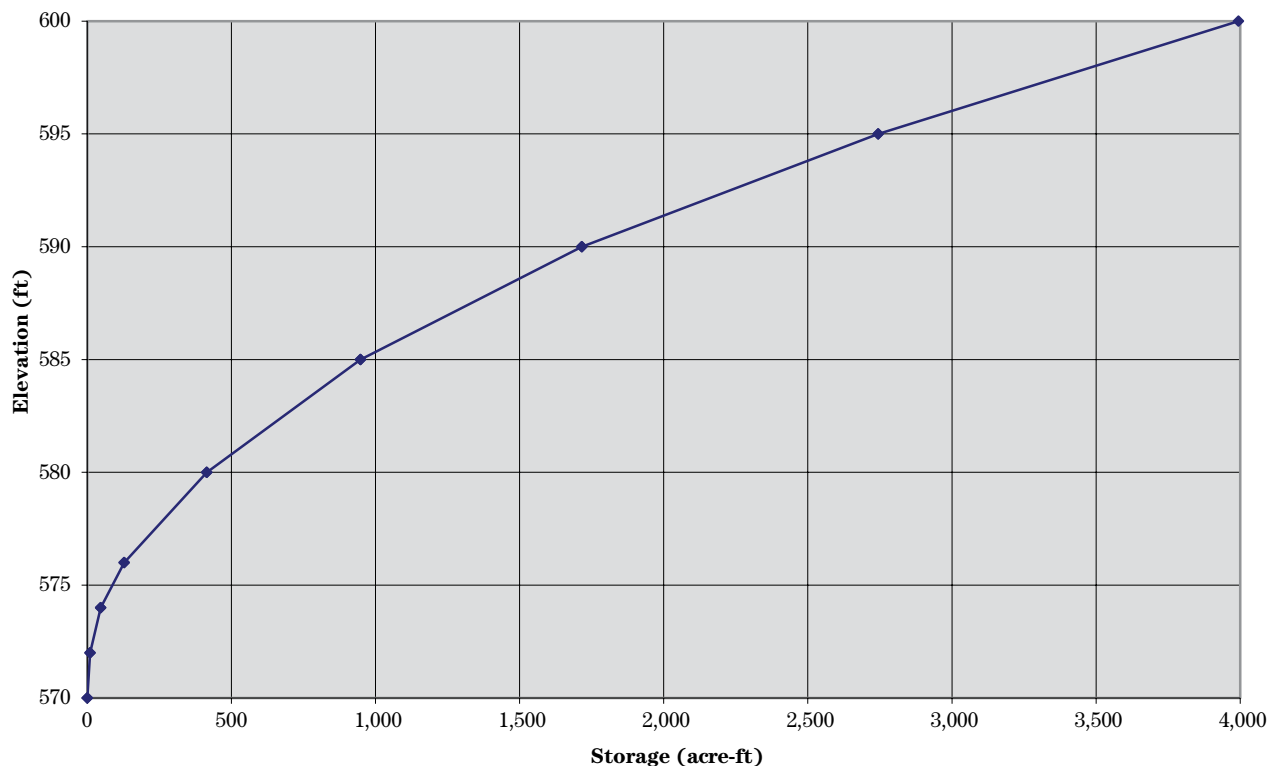
Table 17–10　Principal spillway elevation-discharge data

Elevation	1st stage discharge Weir #1	1st stage discharge Orifice #1	2nd stage discharge Weir #2	2nd stage discharge Orifice #2	Total PS discharge
(ft)	(ft³/s)	(ft³/s)	(ft³/s)	(ft³/s)	(ft³/s)
(1)	(2)	(3)	(4)	(5)	(6)
580.0	0.0				0.0
580.7	4.1				4.1
581.2	11.6				11.6
581.7	21.3				21.3
582.2	32.8				32.8
582.7	45.8				45.8
583.2	60.3	0.0			60.3
583.7	75.3	89.5			75.3
584.2	92.8	101.0			92.8
585.0	122.6	114.6			114.6
586.0	162.0	133.0			133.0
587.0	206.0	149.0	0.0	0.0	149.0
587.5		159.0	44.6	343.0	203.6
588.0		163.0	126.0	347.0	289.0
588.5		170.0	232.0	353.0	353.0
589.0		176.0	357.0	357.0	357.0
589.5		182.0	499.0	361.0	361.0
590.0			656.0	365.0	365.0
590.2			722.0	367.0	367.0
591.0				374.0	374.0
592.0				382.0	382.0
595.0				401.0	401.0
600.0				432.0	432.0

Figure 17–10   Plot of table 17–10, two-stage principal spillway elevation versus total discharge
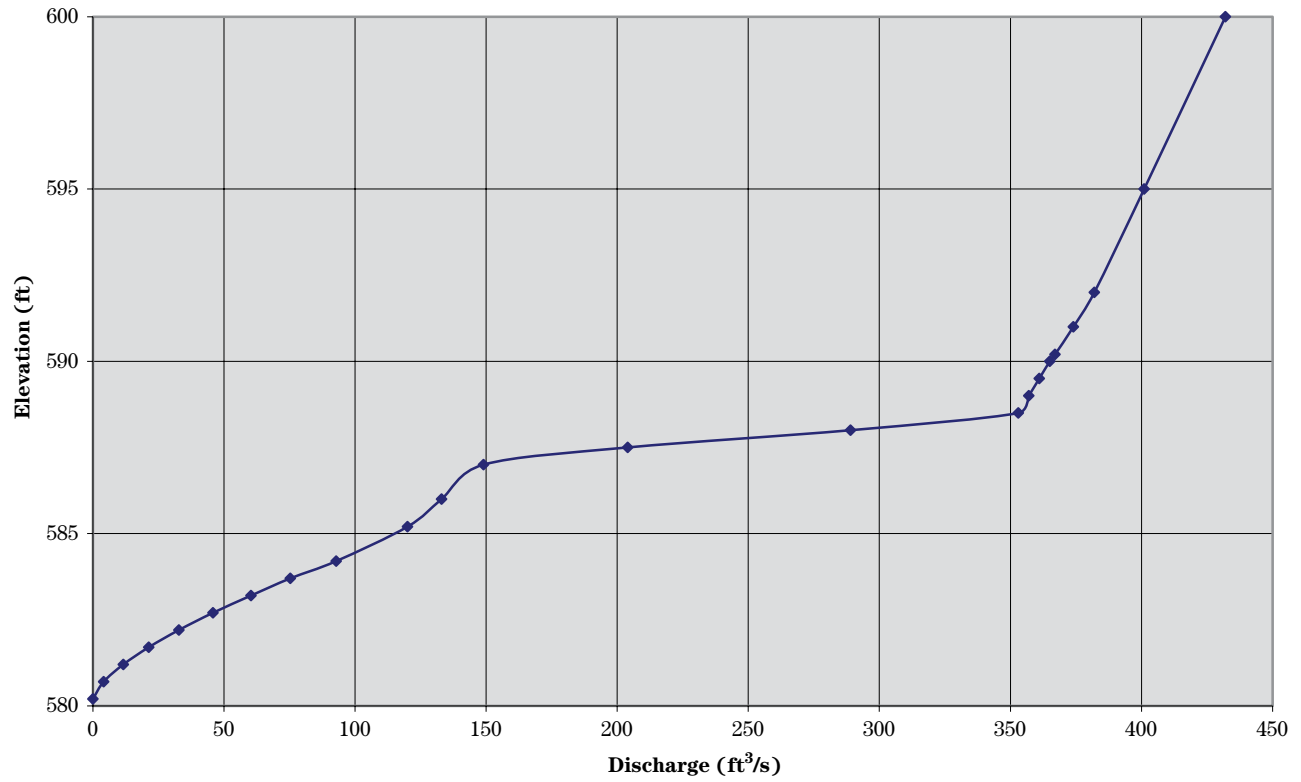
principal spillway flow with the auxiliary spillway flow, a column for the auxiliary spillway discharge would be added between columns 5 and 6, and totals in the final column would include those discharges when appropriate.

English units are used in this example. If SI units are used, elevations would be in meters and discharge values in cubic meters per second.

Step 2: Select the routing time interval— Examination of the PSH in table 17–8 shows two routing intervals will be needed, one of 0.5 day for the small changes in rates occurring in the earlier and later periods of flow, and one of 0.1 day for the large changes immediately before and after the peak discharge. The time interval in days is the difference between consecutive times listed in column 1 of table 17–8.

Step 3: Prepare the working curves in table 17–11. Two working curves are needed because two routing time intervals will be used. Data in columns 1 and 2 of table 17–11 were copied from table 17–10. Values in column 3 were computed by first interpolating the storage from table 17–8 at elevations in column 1 then converting storage in acre-feet to cubic feet per second-days (divide the storage in acre-feet by 1.983) and subtracting 209 cubic feet per second-days to give only the storage available above the first stage principal spillway elevation. Values in column 4 equal the value in column 2 divided by 2 and rounded to whole number. Values in column 5 equal values in column 3 divided by 0.5 day and rounded. The values in column 6 are the sum of values in columns 4 and 5. Values in column 7 equal those in column 3 divided by 0.1 day. The values in column 8 are the

Table 17–11 Working curve data for example 17–2

Elevation	Discharge	Storage	$O_2/2$	For Δt = 0.5 days $S_2/\Delta t$	$S_2/\Delta t + O_2/2$	For Δt = 0.1 days $S_2/\Delta t$	$S_2/\Delta t + O_2/2$
(ft)	(ft³/s)	(ft³/s-days)	(ft³/s)	(ft³/s)	(ft³/s)	(ft³/s)	(ft³/s)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
580.0	0.0	0.0	0	0	0	0	0
581.2	11.6	64.5	6	129	135	645	651
582.2	32.8	118.3	16	237	253	1,183	1,200
583.2	60.3	172.1	30	344	374	1,721	1,751
584.2	92.8	225.9	46	452	498	2,259	2,305
585.0	114.6	268.9	57	538	595	2,689	2,746
587.0	149.0	423.8	75	848	922	4,238	4,312
587.5	203.6	462.5	102	925	1,027	4,625	4,727
588.0	289.0	501.2	145	1,002	1,147	5,012	5,157
588.5	353.0	539.9	177	1,080	1,256	5,399	5,576
590.0	365.0	656.1	183	1,312	1,495	6,561	6,743
592.0	382.0	863.5	191	1,727	1,918	8,635	8,826
595.0	401.0	1,174.5	201	2,349	2,549	11,745	11,945

sum of values in columns 4 and 7. The working curves are plotted in figure 17–11.

The line of equal values plotted in figure 17–11 represents equal values of outflow and $S_2/\Delta t + O_2/2$. For example, at an outflow of 400 cubic feet per second, the value of $S_2/\Delta t + O_2/2$ is 400 cubic feet per second. Note that the line of equal values is well to the left of the working curves; therefore, the routing time intervals are adequately small (see NEH630.1702(f)).

Step 4:   Prepare the operations table, table 17–12. Note that columns 1 and 2 are for time and inflow rate copied from table 17–8. Accumulated times are shown in column 1 of table 17–12 at

intervals of $\Delta t = 0.5$ day for the initial slow-rising portion of the PSH from 0 to 4.5 days, at $\Delta t = 0.1$ day for the fast-rising and falling portion from 4.5 to 6 days, and again at $\Delta t = 0.5$ day for the slow recession from 6 days to the end of the hydrograph.

Step 5:   Enter times and inflows to the operations table. Compute average inflow values. The average inflow values of column 3 are arithmetic averages of entries in column 2.

Step 6:   Do the routing. The procedure is the same as that given in example 17–1 except when a change is made from one working curve to another. Calculations for the first four working curve values (column 4) are shown in the remarks

Figure 17–11   Plot of working curves for example 17–2

Table 17–12 Operations table for example 17–2

Time (days) (1)	Inflow (ft³/s) (2)	Avg. inflow (ft³/s) (3)	$S_2 / \Delta t + O_2 / 2$ (ft³/s) (4)	Outflow (ft³/s) (5)	Remarks (6)
0.0	0	0	0	0	0 + 0 – 0 = 0
0.5	70	35	35	3	35 + 0 – 0 = 35
1.0	79	74	106	9	74 + 35 – 3 = 106
1.5	84	82	179	19	82 + 106 – 9 = 179
2.0	88	86	245	31	
2.5	99	94	308	45	
3.0	110	104	367	59	
3.5	128	119	427	74	
4.0	156	142	495	92	
4.5	245	200	603	115	200 + 495 –92 = 603
4.5	245		2,784	115	Interpolated from table 17–11
4.6	269	257	2,926	119	257 + 2,784 –115 = 2,926
4.7	308	288	3,095	122	
4.8	380	344	3,317	127	
4.9	522	451	3,641	134	
5.0	2,002	1,262	4,769	212	Peak inflow
5.1	1,049	1,526	6,085	358	
5.2	577	813	6,540	363	
5.3	393	485	6,662	364	Peak outflow
5.4	312	352	6,650	364	
5.5	267	290	6,576	363	
5.6	217	242	6,454	362	
5.7	200	208	6,300	360	
5.8	184	192	6,132	359	
5.9	174	179	5,952	357	
6.0	164	169	5,764	355	169 + 5,952 –357 = 5,764
6.0	164		1,295	355	Interpolated from table 17–11
6.5	138	151	1,091	249	151 + 1,295 –355 = 1,091
7.0	118	128	970	174	
7.5	106	112	908	147	
8.0	94	100	860	142	
8.5	88	91	809	137	
9.0	82	85	757	132	
9.5	75	78	703	126	
10.0	67	71	648	120	
Etc.					

(column 6) of table 17–12. At time 4.5 days, the routing time interval changes, therefore, the working curve must be changed. The outflow rate at that time is 115 cubic feet per second. Entering the second working curve with this rate gives 2,784 cubic feet per second as the value of $S_2/\Delta t + O_2/2$ in column 4 for the same time. This value is entered, the routing continues with use of the second working curve. At time 6.0 days, the routing time interval changes back to the first one; and therefore, the first working curve must again be used. The outflow rate at that time is 355 cubic feet per second. Entering the first working curve with this rate gives 1,295 cubic feet per second as the value of $S_2/\Delta t + O_2/2$ in column 4 for that time. After entering this value, the routing continues with use of the first working curve.

Calculations for the working curve values (column 4) where the working curve is changed are shown in the remarks (column 6) of table 17–12. The inflow and outflow hydrographs are plotted in figure 17–12.

Determine the maximum elevation and storage attained in the routing. The elevation and storage may be interpolated from the elevation-discharge and elevation-storage data in table 17–11. The peak outflow is 364 cubic feet per second from table 17–12. Interpolating from table 17–11, the peak elevation is 589.9 feet, and the peak storage is 646.4 cubic feet per second-days. To convert from cubic feet per second-days to acre-feet, multiply the storage in cubic feet per second-days by 1.983 to obtain 1,282 acre-feet.

Figure 17–12   Plot of inflow and outflow hydrographs for example 17–2
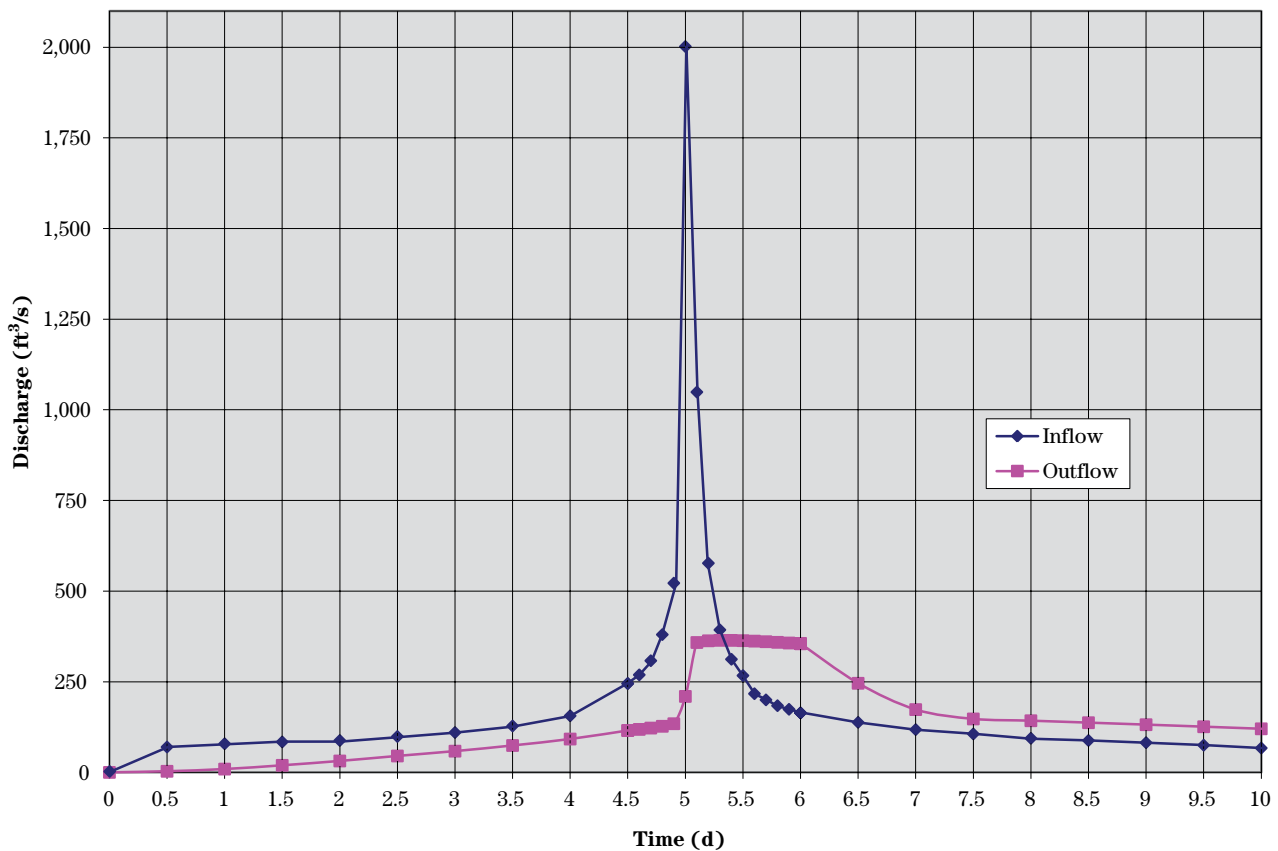
630.1703   Channel flood routing methods

(a)   Hydraulic characteristics of channel reaches

NRCS hydrology computer programs previously used the Convex (SCS 1965) and Modified Att-Kin (Comer, Theurer, and Richardson 1982) channel flood routing methods. The discharge and end-area relation was used to compute one routing coefficient. The use of discharge and end-area values by the Convex and Modified Att-Kin methods allowed the engineer to base the channel routing on as little data as a velocity and reach length. To increase accuracy somewhat, the engineer could enter an elevation-discharge-end-area relation for a reach. This allowed the routing coefficient to be based on a particular discharge. Thus for a given reach, if the hydrograph to be routed remains within the channel, the routing coefficient is based on a channel velocity. If the hydrograph to be routed extends into the floodplain, the routing coefficient is based on an average cross section velocity.

The Muskingum-Cunge (Ponce 1989) method of flood routing uses discharge and end-area along with top width and slope in developing its two routing coefficients. An actual cross section, either surveyed or approximated, is needed for the Muskingum-Cunge method. As a trade-off for requiring more information, greater accuracy and range of applicability are available with the Muskingum-Cunge method. This section outlines the development of hydraulic characteristics of cross sections and channel reaches needed by the Muskingum-Cunge method.

Development of hydraulic information for cross sections and stream reaches is described in detail in NEH630.14, Stage Discharge Relations. Depending on the amount of data available and the purpose of the project, stage discharge relations are generally developed based on water surface profiles for stream reaches using a series of cross sections along a stream or river or by solving Manning's equation for a series of depths at a single cross section. Both of these procedures are automated in the HEC–RAS computer program (U.S. Army Corps of Engineers 2010).

The NRCS hydrologic models WinTR–55, SITES, and WinTR–20 are set up to assign a single cross section to represent the hydraulic characteristics of a particular reach.

Two important judgments are needed by the engineer when developing data for channel reach routing. One is the selection of a representative cross section. This cross section should represent an average flow velocity through the reach. If there are several cross sections available for selection, the engineer should select the section most typical of the reach conditions. The second important judgment is the selection of a reach length. To properly represent reach storage characteristics, the reach length should be an average length over the routing reach. HEC–RAS allows for left overbank, right overbank, and channel reach lengths. WinTR–20 allows for only floodplain length and channel length. WinTR–55 is limited to only a single channel length.

Storage in a reach is often underestimated because backwater storage in tributaries is usually not considered in developing water surface profiles using HEC–RAS. If this type of storage is significant, it should be estimated. The simplest way to account for this would be to increase the floodplain length. A more complex analysis would involve adjusting HEC–RAS cross sections to include an ineffective flow area for tributary backwater.

(b)   Muskingum method

A brief review of the Muskingum method precedes the description of the Muskingum-Cunge method. The routing equation is similar for the methods. The Muskingum channel routing method is based on two equations (Linsley, Kohler, and Paulhus 1982). The first is the continuity equation or conservation of mass.

$$\frac{\left(I_1 + I_2\right)}{2}\Delta t - \frac{\left(O_1 + O_2\right)}{2}\Delta t = S_2 - S_1 \qquad \text{(eq. 17–5)}$$

Equation 17–5 states that inflow to the reach minus outflow from the reach is equal to the change in storage. This is the same basic equation used in routing of reservoirs. Where the reservoir routing method assumes a relation of storage and outflow discharge, the Muskingum method assumes the amount of storage is related to both inflow and outflow discharge. The

reservoir routing method assumes a level pool, and the Muskingum method assumes a sloping water surface.

$$S = K\left\{XI + (1-X)O\right\} \qquad \text{(eq. 17–6)}$$

where :
S   = reach storage, ft^3
K   = storage constant, s
X   = weighting factor, dimensionless
I   = inflow discharge, ft^3/s
O   = outflow discharge, ft^3/s

When X equals zero in equation 17–6, the equation reduces to a simple relation of storage and outflow discharge: S is KO (reservoir routing assumption). The values in the equations are in units of feet and seconds. They may also be defined in any units of length and time as long as all values in the equation are consistent.

Combining equations 17–5 and 17–6 and simplifying results in the following equation (Ponce 1981; Linsley, Kohler, and Paulus 1982):

$$O_2 = C_1 I_1 + C_2 I_2 + C_3 O_1 \qquad \text{(eq. 17–7)}$$

where:

$$C_1 = \frac{\left[\left(\dfrac{\Delta t}{K}\right) + 2X\right]}{C_0} \qquad \text{(eq. 17–8)}$$

$$C_2 = \frac{\left[\left(\dfrac{\Delta t}{K}\right) - 2X\right]}{C_0} \qquad \text{(eq. 17–9)}$$

$$C_3 = \frac{\left[2(1-X) - \dfrac{\Delta t}{K}\right]}{C_0} \qquad \text{(eq. 17–10)}$$

$$C_0 = \frac{\Delta t}{K} + 2(1-X) \qquad \text{(eq. 17–11)}$$

C_0, C_1, C_2, and C_3 are dimensionless coefficients with the sum of C_1, C_2, and C_3 equal to 1.0.

An approximation for K is the travel time through the reach or the length of reach divided by the average flow velocity. Either water surface profiles or a solution of Manning's equation are needed for the reach to

estimate the average flow velocity. The approximation of K is sensitive to the value of discharge at which velocity is selected. The approximation of K is also sensitive to whether the channel length, floodplain length, or some type of weighted reach length is used. The value of X is between 0.0 and 0.5. A value of 0.0 gives maximum attenuation from the procedure, and 0.5 provides the minimum attenuation. The value of X is difficult to estimate without stream gage information. Linsley, Kohler, and Paulhus (1982) and other hydrology texts describe a procedure to determine K and X from stream hydrographs.

(c) Muskingum-Cunge method

Cunge (1969) developed equations to estimate K and X from hydraulic properties of the reach. These are based on the linking of the convection diffusion model and the Muskingum routing equation. A condensed derivation is described.

The one-dimensional St. Venant equations for hydraulic routing describe the movement of a flood wave. Equation 17–12 is the continuity equation describing the conservation of mass. Equation 17–13 describes the conservation of momentum.

$$\frac{\partial d}{\partial t} + \frac{\partial(Vd)}{\partial x} = 0 \qquad \text{(eq. 17–12)}$$

$$\frac{\partial V}{g\partial t} + \frac{V\partial V}{g\partial x} + \frac{\partial d}{\partial x} = S_o - S_f \qquad \text{(eq. 17–13)}$$

where:
∂ = the partial derivative operator
V   = velocity of discharge or flow, ft/s
d   = depth of flow, ft
t   = time, s
x   = distance, ft
g   = acceleration of gravity, ft/s^2
S_o = slope of channel in longitudinal direction, ft/ft
S_f = friction slope, ft/ft

The first two terms of equation 17–13 represent acceleration of the water. Studies of the magnitudes of the various terms have shown when it is important to include them in an analysis of routing of hydrographs in stream networks. When the slope of the channel and friction slope are very close with a small difference in values, the first two terms on the left side of the equa-

tion are either very small or cancel each other out due to positive and negative signs. In these cases, neglecting these first two terms does not significantly affect accuracy of the routing.

When there is a larger difference between the channel and friction slopes, such as in very flat channel gradients, the terms on the left side of the equation are significant. The acceleration terms represent the change of velocity with time and change of velocity with distance. These terms are most significant with rapidly rising hydrographs typical of a breaching dam or a flash flood.

Traditional hydrologic routing techniques typically ignore all three terms on the left side of equation 17–13. When the first two terms on the left side are neglected, equation 17–13 becomes what is called a diffusion analogy. The Muskingum-Cunge routing method is derived from this simplified equation in addition to the conservation of mass equation, equation 17–12. Since the Muskingum-Cunge routing method includes one more term in the momentum equation, it should be expected to provide more accurate routing results and be applicable over a wider range of conditions. In comparing the results with the solution of the St. Venant equations, this has shown to be the case.

The mathematical derivation presented by Ponce (1981) is condensed here. It is repeated for background on the development of the Muskingum-Cunge routing procedure. The model is based on setting the diffusivity of the numerical model equal to the diffusivity of the theoretical model.

The theoretical model is based on the continuity equation and the conservation of momentum equation (eq. 17–13) with the first two terms removed.

$$\frac{1}{T}\left(\frac{\partial Q}{\partial x}\right) + \frac{\partial d}{\partial t} = 0 \qquad \text{(eq. 17–14)}$$

$$\frac{\partial d}{\partial x} + S_f - S_o = 0 \qquad \text{(eq. 17–15)}$$

where:
T   = top width, ft

The diffusivity (μ) of the theoretical model is:

$$\mu = \frac{Q}{2TS_o} \qquad \text{(eq. 17–16)}$$

where:
μ = theoretical diffusivity, ft^2/s

The numerical model is formulated for solution on a routing distance and time step grid (fig. 17–13).

$$\frac{X(I_2 - I_1) + (1-X)(O_2 - O_1)}{\Delta t} + c\,\frac{(O_1 - I_1) + (O_2 - I_2)}{2\Delta x} = 0$$
$$\text{(eq. 17–17)}$$

where:
c   = flood wave celerity, ft/s
Δx = routing distance step, ft

The diffusivity of the numerical model is:

$$\mu_n = c(\Delta x)\left(\frac{1}{2} - X\right) \qquad \text{(eq. 17–18)}$$

where:
μ_n = numerical diffusivity, ft^2/s.

Assume that the two diffusivities are equal:

$$c(\Delta x)\left(\frac{1}{2} - X\right) = \frac{Q}{2TS_o} \qquad \text{(eq. 17–19)}$$

From which:

$$X = \frac{1}{2}\left(1 - \frac{Q}{\left[T(S_o)(c)\Delta x\right]}\right) \qquad \text{(eq. 17–20)}$$

Figure 17–13   Muskingum-Cunge routing distance and time step solution grid
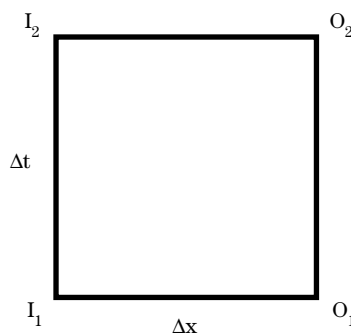
The routing distance step, Δx, is estimated from hydraulic characteristics of the reach. A reach is represented by a single cross section in WinTR–20. This emphasizes the importance of using a cross section representative of the reach. If the reach is such that a single cross section cannot represent it, the reach may be divided into shorter reaches where a single cross section is more representative of each routing distance step.

The equation for a routing distance step is:

$$\Delta x = 0.5 \left(c\Delta t + \frac{Q}{T\, S_o\, c} \right) \qquad \text{(eq. 17–21)}$$

If Δx is greater than the routing reach length, L, the reach is not divided into steps. If Δx is between 1/2 L and L, the reach is divided into two steps. If Δx is between 1/3 L and 1/2 L, there are three routing distance steps. If Δx is between 1/4 L and 1/3 L, there are four routing distance steps. If Δx is between 1/5 L and 1/4 L, there are five routing distance steps.

The number of routing distance steps in a routing reach is determined within WinTR–20 and WinTR–55. Detailed information on routing distance steps and routing coefficients may be requested in the WinTR–20 output. More explanation of routing distance steps is contained in appendix C.

The storage constant for the Muskingum-Cunge method is:

$$K = \frac{\Delta x}{c} \qquad \text{(eq. 17–22)}$$

With the values of X and K determined based on cross section and reach hydraulic properties, the routing coefficients in equations 17–8 to 17–11 are computed. The routing equation (equation 17–7) is solved for the reach for each time and routing distance step. The outflow at time step 2 (O_2) is solved using known discharges I_1, I_2, and O_1 (fig. 17–13).

(d) Compare and contrast the Muskingum and Muskingum-Cunge routing methods

The Muskingum-Cunge method is more applicable for use on ungaged streams. It uses the same routing equation as the original Muskingum method. The weighting factor X is defined mathematically in the Muskingum-Cunge method and is based on hydraulic properties of the reach. The solution technique is somewhat different in that the Muskingum method treats the reach as a single routing distance step and the Muskingum-Cunge method defines a routing distance step length and sub-steps a reach if needed (appendix C).

The storage constant K is defined differently in the two methods. The Muskingum method defines K in the relation of discharge and storage, and the Muskingum-Cunge method defines K as the routing distance step length divided by the wave celerity. K has the units of time in both procedures and X is nondimensional. The value of X in the Muskingum method may vary from 0.0 to 0.5. The value of X for the Muskingum-Cunge may be positive or negative.

There are several formulations of the Muskingum-Cunge method based on the way coefficients and routing distance steps are determined and in the way the routing equation is solved in the routing distance and time discretization. The formulation in WinTR–20 is a constant coefficient method in that the routing coefficients C_1, C_2, and C_3 are constant for a routing distance step. The routing equations as formulated conserve hydrograph volume so that the inflow volume to the reach is equal to outflow volume from the reach. Lateral inflow to the reach is not included in the routing equations. Lateral inflow may be added in WinTR–20 at the upper and/or lower end of a reach. The solution to the routing equation in WinTR–20 is a direct solution technique without computational iterations because the outflow at time 2 is solved based on the three known discharges: inflow at times 1 and 2, and outflow at time 1.

(e) Definition and significance of *m*

The discharge versus end-area relation for simple cross sections (rectangular, triangular, or trapezoidal) may be fit by a power curve function of the form:

$$Q = x A^m \qquad \text{(eq. 17–23)}$$

where:
x = coefficient,
m = exponent

The exponent m has a physical meaning which can add to the understanding of the Muskingum-Cunge routing method. According to hydraulic theory, the speed at which a flood wave travels downstream is called the celerity and is equal to the slope of the discharge versus end-area curve at a given discharge. In equation form:

$$c = \frac{dQ}{dA} \qquad \text{(eq. 17–24)}$$

where:
d   = derivative operator

Combining equations 17–23 and 17–24 and differentiating with respect to end-area A results in:

$$c = dQ/dA = xmA^{(m-1)} = m(xA^m)/A = mQ/A$$
$$\text{(eq. 17–25)}$$

$$c = mV \qquad \text{(eq. 17–26)}$$

The exponent m is therefore a factor relating average velocity and the wave velocity or celerity. The wave velocity is important to the Muskingum-Cunge routing method in computation of the Muskingum-Cunge routing coefficients for the reach. For details on routing equations see NEH630.1703(b) and (c).

The value of m used in the routing has a major effect on the calculation of travel time of the hydrograph through a reach. The exponent m is inversely related to the travel time. Therefore the selection of m can affect the timing of peaks in any reach and can affect the way in which tributary hydrographs add to the peak of the hydrograph on the main stream.

The WinTR–20 program has internally set limits on the value of m used in reach routings. If the m is too large or too small there is a weakness in both the mathematical and numeric models. A value of m less than 1.0 is unrealistic because the wave velocity or celerity is less than the average flow velocity. Therefore, the m value used in any given routing is not allowed to be less than 1.0.

The following derivation shows how m may be calculated from Manning's equation.

The familiar Manning's equation for discharge may be rearranged to:

$$Q = \frac{1.49 S_f^{1/2}}{n p^{2/3}} A^{5/3} \qquad \text{(eq. 17–27)}$$

where:
n = Manning's roughness coefficient
p   = wetted perimeter, ft

NEH630.14, Stage Discharge Relations (2012), contains appendix A on estimating the value of Manning's n, and many hydraulics texts have tables giving values of n in various situations (Brater and King 1982).

For cross sections where the hydraulic radius (A/p) can be approximated by the depth as with a wide channel with top width, T, equation 17–27 takes the form:

$$Q = xA^m \qquad \text{(eq. 17–28)}$$

where:

$$x = \frac{\left(1.49 S_f^{\frac{1}{2}}\right)}{n T^{\frac{2}{3}}} \text{ and } m = \frac{5}{3}$$

The description concerning celerity and Manning's equation dealt with simple cross section shapes, but is also applicable to most natural channel cross sections. This concept does not apply directly to cross sections with both channel and floodplain portions. Discharge-end-area plots for these types of cross sections generally exhibit changes in slope and attempting to fit one power curve to the data can result in significant differences between the original plot and the fitted curve.

If the slope of the discharge-end-area curve plotted on log-log paper is greater than one, the average velocity at the cross section is increasing with increasing discharge. If the slope is less than one, average velocity is decreasing with increasing discharge. The third possibility is that the slope equals one and the velocity is constant regardless of discharge.

In a floodplain cross section, Manning's n is generally variable laterally as the Manning's n in the channel is different from that in the floodplain. The word "segment" refers to a portion of a cross section with one assigned value of Manning's n. For any single segment

of a multisegment cross section, the average velocity should increase with increasing discharge. However, when adding the discharge and end-area for all segments to get the total discharge and total end-area at a cross section, there can be elevations where the average velocity actually decreases as flow increases. The most common occurrence of this is at discharges exceeding channel capacity with shallow depth in the floodplain for which the velocity in the channel is much larger than the velocity in the floodplain.

The slope of the discharge-end-area curve where the average velocity decreases (which can lead to a celerity significantly less than one) does not represent the true celerity or wave propagation speed. For this reason, a procedure to calculate m at a cross section was developed.

(f)   Calculation of *m* from rating table data

A rating table to represent flow in a reach is required to complete a Muskingum-Cunge routing in WinTR–20. Data required include elevation, discharge, end-area, top width, and friction slope for a minimum of three discharges (two greater than zero). All parameters except friction slope must increase from one value to the next in the table. If any value of elevation, flow, end-area, or top width is the same or decreasing, there will be an error message when using WinTR–20.

A value of m is computed at each discharge of the rating table. This procedure will reflect changes in slope of the discharge versus end-area curve typically occurring above the bankfull discharge and at large floodplain discharges.

The equations used to compute m are:

$$m = S(2,3) \qquad \text{(eq. 17–29a)}$$

and

$$Q(I) < Q(3) \qquad \text{(eq. 17–29b)}$$

where:
S(2,3) = log-log slope of discharge-end-area curve between points 2 and 3
Q(I)   = discharge at flow number I, ft³/s

$$m(I) = \frac{\left(Q(3)S(2,3) + \sum \left(Q(I) - Q(I-1) \right) S(I-1,I) \right)}{Q(I)}$$

$$\text{(eq. 17–30a)}$$

where the summation goes from $I=4$ to the number of elevation points in the rating table and

m(I)   = m at flow number I
S(I–1, I) = log-log slope of discharge-end-area curve between points I–1 and I

For:

$$Q(3) < Q \le Q(I) \qquad \text{(eq. 17–30b)}$$

When calculated this way m can be considered a weighted slope.

When m is desired at a non-tabulated point, m is interpolated on a log-log basis.

If the log-log slope between each two consecutive points in a rating table is the same, m will be the same for all discharges. In other words, one power curve will fit the data accurately. When the log-log slope of the discharge-end-area curve changes, m will change. The value of m directly impacts the values of the storage constant K and the weighting factor X used in the Muskingum-Cunge routing procedure.

An example of calculation of m for a rating curve is in table 17–13.

Based on cross section properties, channel capacity is 500 cubic feet per second. At discharges higher than 500 cubic feet per second, the average velocity reduces due to low floodplain velocity. At discharges above 870 cubic feet per second, the average velocity increases again with deeper flow in the floodplain. The slope (log-log) between each pair of consecutive points is calculated:

$$S(2,3) = \frac{\log\left(\dfrac{500}{200}\right)}{\log\left(\dfrac{200}{110}\right)} = 1.5$$

$$S(3,4) = \frac{\log\left(\dfrac{870}{500}\right)}{\log\left(\dfrac{800}{200}\right)} = 0.4$$

$$S(4,5) = \frac{\log\left(\dfrac{1500}{870}\right)}{\log\left(\dfrac{1200}{800}\right)} = 1.3$$

For all discharges at or below 500 ft³/s, m is 1.5.

At a discharge of 870 ft³/s,

$$m = \frac{\left(500(1.5)\right) + \left((870 - 500)0.4\right)}{870}$$
$$= 1.03$$

At a discharge of 1,500 cubic feet per second,

$$m = \frac{\left(500(1.5)\right) + \left((870 - 500)0.4\right) + \left((1{,}500 - 870)1.3\right)}{1{,}500}$$
$$= 1.14$$

At a nontabulated discharge, such as 1,000 cubic feet per second, the value of m is interpolated between values of m at the higher and lower tabulated discharges on a log-log basis.

Table 17–13   Calculation of m for a rating curve

Discharge (ft³/s)	End-area (ft²)	Slope, S (%)	m
0	0	—	1.5
200	110	—	1.5
500	200	1.5	1.5
870	800	0.4	1.03
1,500	1,200	1.3	1.14

(g) Steps in routing by the Muskingum-Cunge method

The steps involved in application of the Muskingum-Cunge method in WinTR–20 are outlined next. The purpose is to allow for understanding of the procedure, its assumptions and limitations, and checking the results of the WinTR–20 program.

Step 1: Find the rise time of the peak of the inflow hydrograph to see if Δt of the inflow hydrograph needs to be reduced. This is defined in WinTR–20 as the difference between two specified times, T_1 and T_2. T_1 is the time of the hydrograph point immediately before the one where the discharge exceeds 5 percent of the peak discharge of the inflow hydrograph. T_2 is the time to the peak discharge.

In the case of multipeak hydrographs, T_2 is the time of the first peak in the hydrograph that exceeds 50 percent of the highest peak discharge of the hydrograph. If there are less than 10 time intervals between T_1 and T_2, the hydrograph is interpolated at a new time interval such that there are 10 intervals between times T_1 and T_2. This time interval is used throughout the reach routing.

At the end of the reach routing, the hydrograph is interpolated back to the original time interval. The reason for this refinement of the time interval is that when the rise of the hydrograph occurs in only a few time steps, the results of the Muskingum-Cunge routing exhibit instability, which is unacceptable.

Step 2: Determine the end-area, top width, m, and friction slope at the reference discharge. The reference discharge is defined as the peak discharge of the inflow hydrograph. The Muskingum-Cunge as formulated in WinTR–20 is a constant coefficient routing. This means that for a routing distance step, the coefficients remain fixed. If there is more than one routing distance step, the reference discharge is the peak of the inflow hydrograph at the upstream end of each routing distance step.

Step 3: Calculate the reference celerity, which is equal to m multiplied by the reference discharge divided by the reference end-area.

$$c_r = m \frac{Q_{pi}}{A_r}$$ (eq. 17–31)

where:

c_r = reference wave celerity (corresponding to peak of inflow hydrograph), ft/s

Q_{pi} = peak discharge of the inflow hydrograph, reference discharge, ft^3/s

A_r = cross section end-area at the reference discharge, ft^2

m = exponent

Step 4:   Calculate Δx (the routing distance step length). If Δx is greater than the reach length, there is only one routing distance step. If the value of Δx is less than the reach length, the reach is routed in two or more steps. This is determined by dividing the reach length by the routing distance step length and rounding this ratio to the next higher integer number. For example, if the reach length is 8,100 feet and Δx is 3,000 feet, the ratio of length to Δx is 2.7. Round 2.7 up to 3. There would be 3 routing distance steps of 2,700 feet each. Actually, Δx could be anywhere between 2,700 and 4,049 feet, and there would still be 3 routing distance steps (the ratio of reach length to Δx rarely falls exactly on a whole number such as 3.0; but when it does, that is the number of routing distance steps).

Step 5:   The reference end-area, m, reference top width, and reference friction slope are based on the reference discharge (the peak discharge at the upper end of the routing distance step). The reference celerity, the storage constant K, the weighting factor X, and the routing coefficients C_0, C_1, C_2, and C_3 are calculated using equations 17–8 through 17–11, 17–20, and 17–22.

Step 6:   Compute the Courant number and the grid Reynolds number. The Courant number C_g is the ratio of the physical wave celerity to the grid celerity Δx/Δt (Ponce 1989). These values are computed to determine if the routing is within accuracy limits of the Muskingum-Cunge method (see appendix 17–B for more details).

These coefficients are nondimensional and are defined as:

$$C_g = c_r (3,600) \frac{\Delta t}{\Delta x}$$ (eq. 17–32)

where:

C_g = Courant number, the ratio of the physical wave celerity to the grid celerity Δx/Δt, dimensionless

Since K is defined in equation 17–22:

$$K = \frac{\Delta x}{c}$$

A simpler equation for the Courant number is:

$$C_g = \Delta t \times \frac{3,600}{K}$$ (eq. 17–33)

The grid Reynold's number D_g is the mathematical criterion that distinguishes laminar from turbulent flow.

$$D_g = \frac{Q_{pi}}{T_r S_r c_r \Delta x}$$ (e–q. 17–34)

where:

D_g = Grid Reynold's number, a mathematical criterion which distinguishes laminar from turbulent flow, dimensionless

T_r = reference top width of cross section at the peak discharge of the inflow hydrograph, feet

S_r = reference friction slope at the peak discharge of the inflow hydrograph, ft/ft

Since X is defined in equation 17–20 as:

$$X = \frac{1}{2} \left(1 - \frac{Q}{\left[T(S_o)(c) \Delta x \right]} \right)$$

A simpler equation for Grid Reynolds number (D_g) is:

$$D_g = 1 - 2X$$ (eq. 17–35)

$$D_g (\text{critical}) = e^{(2.3 C_g)}$$ (eq. 17–36)

where:

e = Base of natural logarithms, or approximately 2.71828

If D_g is less than D_g (critical), the routing conditions are within the accuracy limits. If D_g is greater than D_g (critical), the routing conditions are outside the accuracy limits.

Step 7: The routing equations are next solved for each routing distance step using the following procedure. The outflow O_2 is determined by solving the routing equation (eq. 17–7) for each time step starting with known values of I_1, I_2, and O_1. Repeat steps 2 through 6 (but not step 3) for each routing distance step if there are more than one routing distance steps. Step 3 is not repeated because the routing distance step and number of routing distance steps have already been determined.

When all the routing distance steps have been completed, the reach routing is completed. When running WinTR–20, the user may obtain intermediate results for the reach routing such as values of routing coefficients and hydrographs at each routing distance step.

(h) Use of the Muskingum-Cunge method with meandering channels

The effect of a meandering channel on flood routing can be significant depending on the size of the channel and degree of meandering. When computing storage in the reach, the cross section end-area and reach length are routinely multiplied. In a meandering channel, the channel length can be significantly longer than the length of the floodplain, which will result in more volume stored in the reach and more peak discharge attenuation. A procedure has been developed to account for this additional storage if the user feels it is significant. The WinTR–20 computer program uses this procedure.

Information needed to use this procedure includes a measurement of channel length and floodplain length for the reach and a rating table consisting of several elevations with associated total cross section discharge and total cross section end-area. Total is interpreted to mean the sum of both the channel and floodplain values.

The procedure is based on use of the channel length multiplied by the channel end-area to obtain channel storage and use of the floodplain length and floodplain end-area to obtain floodplain storage. These storages are then added to get total reach storage. An elevation must be selected based on cross section information, which will be the dividing line between channel and floodplain end-area. This is defined as the "low ground" elevation.

In an ideal cross section where both channel banks are at the same elevation and the floodplain elevations are all at or above the bank elevation, the channel bank elevation is the obvious selection as the "low ground" elevation. In actual cross sections, the elevation to select is not so obvious. Examination of a cross section plot is often useful. Sometimes the low point of the floodplain is outside the channel banks. This is usually the elevation where the floodplain end-area begins to increase and overshadow the channel end-area. This elevation is recommended unless the cross section plot indicates there is a better dividing elevation. Figure 17–14 shows an example of a cross section and the selection of the "low ground" elevation.

The water level for any profile in the cross section of figure 17–14 is assumed to be horizontal (the same elevation in the channel and both floodplains). If this is not the case, a more complex hydraulic analysis may be needed. which may include an unsteady flow routing (HEC–RAS, for example).

Example 17–3 End-area calculation for meandering channel
A reach has a channel length of 6,000 feet and a floodplain length of 5,000 feet. The meander factor is 6,000/5,000 or 1.2. The "low ground" elevation at the cross section is 403. Table 17–14 is the rating table for example 17–3.

Table 17–15 shows computation results. The object of the table is to develop a net end-area which when multiplied by the channel length will give the correct reach storage at a particular elevation in the cross section.

The floodplain end-area is the difference in total end-area (A) and channel end-area. The column for revised floodplain end-area is the floodplain end-area divided by the meander factor (1.2). The net end-area column is the sum of the channel end-area and the revised floodplain end-area columns.

Figure 17–14   Cross section with "low ground" elevation identified
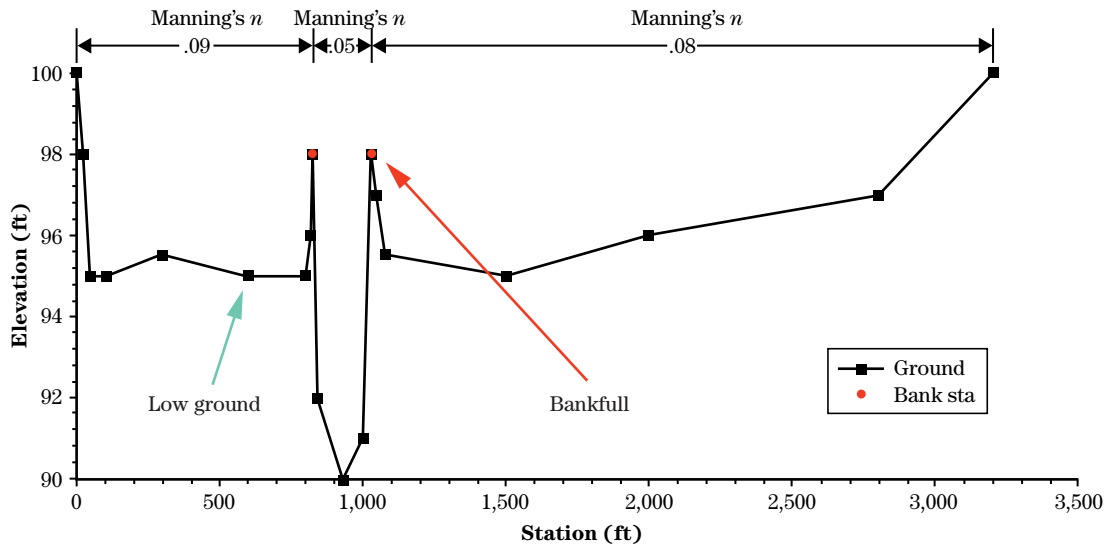
Table 17–14   Rating table for example 17–3

Elevation (ft)	Discharge (Q, ft³/s)	End-area (A, ft²)
400	0	0
401	10	3
402	25	8
403	50	15
404	100	30
405	200	75
406	400	125
407	600	175

When completing the flood routing for this reach using the Muskingum-Cunge method, the elevation, discharge, and net end-area columns of this table should be used in conjunction with the channel length to represent the reach length.

Example 17–4   Muskingum-Cunge routing of a channel cross section.

This example shows the procedure to route a hydrograph through a channel reach. The channel and reach properties are: bottom width is 15 feet, depth is 8 feet, side slope is 2:1, reach slope is 0.00091 foot per foot, Manning's n is 0.04, and reach length is 2,600 feet.

HEC–RAS was used to develop a relation of elevation, discharge, area, top width, and slope. Manual methods included in NEH630.14 (2012) could also have been used. Table 17–16 includes a complete cross section with channel and floodplain. Example 17–5 will route a hydrograph that will utilize the floodplain. The table also includes the value of m (weighted log-log slope of the discharge versus area curve) at each elevation.

The inflow hydrograph has been developed for the watershed upstream of the reach (fig. 17–15).

The inflow hydrograph is defined as a table with a constant time interval. In this example, the time interval is 0.1 hour. The hydrograph is shown in table 17–17 in tabular form.

Step 1:   Find the number of time intervals to the peak discharge in table 17–17. The peak discharge of the inflow hydrograph is 684.18 at time 12.9 hours. Five percent of the peak is 34 cubic feet per second, which occurs about 11.7 hours. At a time interval of 0.1 hour, there are 12 intervals to the peak. Since this is more than 10, the inflow hydrograph does not need to be interpolated at a smaller time increment.

Step 2:   Compute parameters for the reference peak discharge of the inflow hydrograph. The reference discharge is 684.18 cubic feet per second. The routing coefficients are calculated based on hydraulic parameters of the peak discharge. Interpolate the reference end-area, reference top width, reference friction slope and m from table 17–16 at the reference discharge of 684.18 cubic feet per second. The reference end-area is 220.9 square feet, the reference top width is 44.6 feet, the reference friction slope is 0.00091, and m is 1.3873.

Compute the reference wave celerity using equation 17–31:

$$c_r = 1.3873 \frac{684.18}{220.9}$$
$$= 4.297 \text{ ft/s}$$

Table 17–15   Rating table computation results for example 17–3

Elevation (ft)	Q (ft³/s)	A (ft²)	Channel end-area (ft²)	Floodplain end-area (ft²)	Revised floodplain end-area (ft²)	Net end-area (ft²)
400	0	0	0	0	0	0
401	10	3	3	0	0	3
402	25	8	8	0	0	8
403	50	15	15	0	0	15
404	100	30	15	15	12.5	27.5
405	200	75	15	60	50	65
406	400	125	15	110	91.7	106.7
407	600	175	15	160	133.3	148.3

Table 17–16 Cross section hydraulic characteristics for examples 17–4 and 17–5

Elevation (ft)	Discharge (ft³/s)	End-area (ft²)	Top width (ft)	Slope (ft/ft)	m
90.00	0	0.00	0.00	0.00091	1.424
92.40	80	47.43	24.59	0.00091	1.424
93.51	160	77.16	29.02	0.00091	1.424
94.35	240	103.11	32.40	0.00091	1.416
95.05	320	126.91	35.22	0.00091	1.408
95.67	400	149.21	37.67	0.00091	1.402
96.21	480	170.42	39.85	0.00091	1.397
96.71	560	190.74	41.84	0.00091	1.393
97.17	640	210.36	43.68	0.00091	1.389
97.60	720	229.41	45.39	0.00091	1.386
98.00	800	247.87	46.99	0.00091	1.384
98.35	900	314.26	335.53	0.00091	1.285
98.70	1,050	484.33	629.18	0.00091	1.152
99.19	1,400	806.77	673.22	0.00091	1.005
99.79	2,000	1,219.12	697.29	0.00091	0.963
100.20	2,500	1,508.75	713.71	0.00091	0.980
100.56	3,000	1,769.29	728.17	0.00091	1.007
100.95	3,600	2,059.90	743.96	0.00091	1.039
102.01	5,500	2,867.85	786.5	0.00091	1.123
103.73	9,450	4,281.57	855	0.00091	1.218

Figure 17–15 Plot of inflow hydrograph for example 17–4
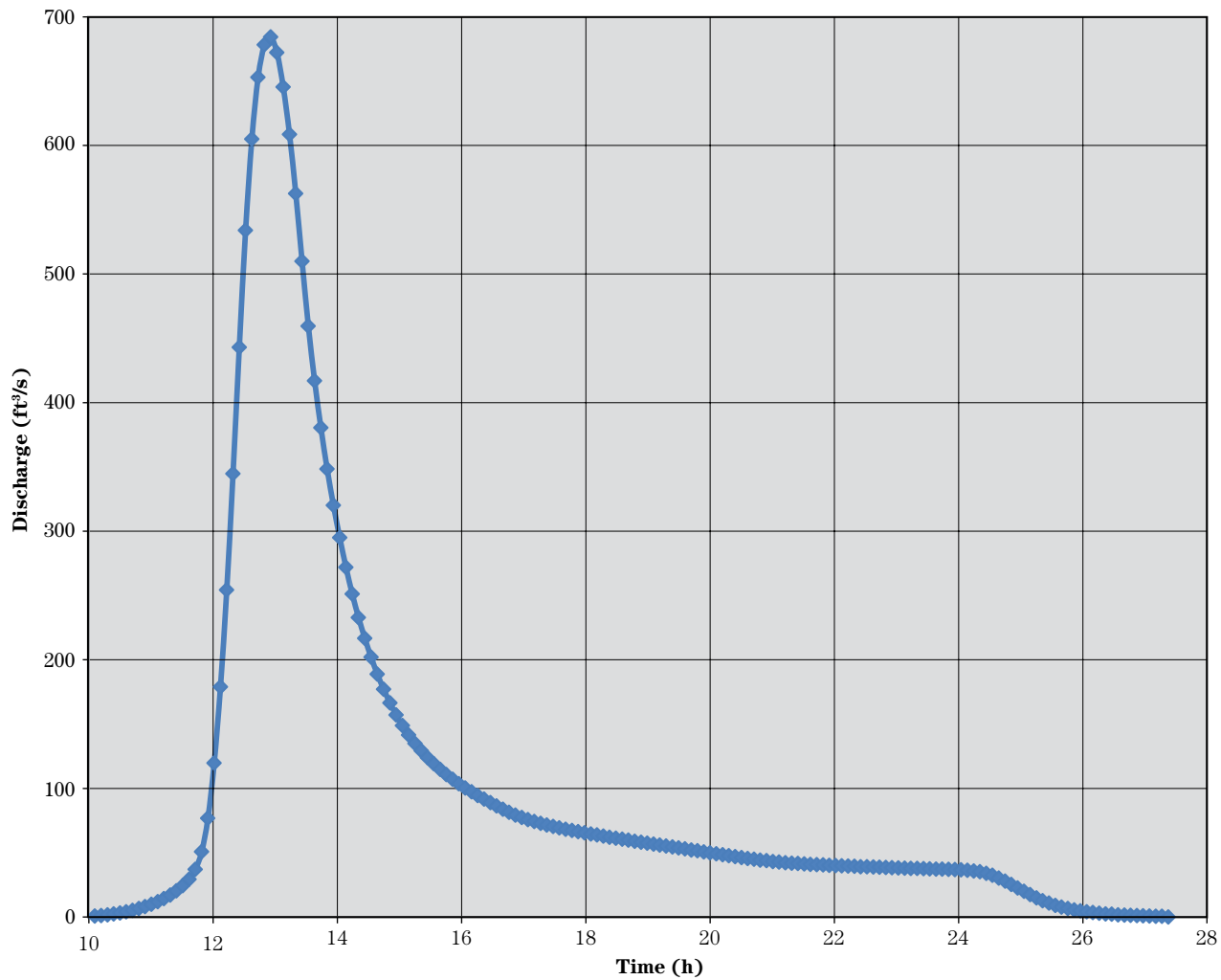
Table 17–17　Inflow hydrograph for example 17–4

Time (h)	Discharge (ft³/s)	Discharge (ft³/s)	Discharge (ft³/s)	Discharge (ft³/s)	Discharge (ft³/s)	Discharge (ft³/s)	Discharge (ft³/s)
10.1	0.7	1.1	1.6	2.3	3.1	4.0	5.1
10.8	6.5	8.0	9.8	11.9	14.3	17.1	20.4
11.5	24.3	29.3	37.0	50.7	76.7	119.7	179.0
12.2	254.2	344.7	443	533.8	604.9	652.9	678.2
12.9	684.18	672.1	645.3	608.6	562.4	509.9	459.5
13.6	416.9	380.5	348.3	320.1	295	271.9	251.1
14.3	232.9	216.7	202.0	188.8	177.1	166.5	157.1
15.0	148.9	141.6	135	129.2	123.9	119.1	114.7
15.8	110.7	107.0	103.5	100.3	97.3	94.4	91.6
16.5	88.9	86.3	83.7	81.3	79.2	77.3	75.7
17.2	74.2	72.8	71.5	70.4	69.3	68.2	67.3
17.9	66.3	65.4	64.5	63.7	62.9	62.0	61.2
18.6	60.5	59.7	58.9	58.1	57.4	56.6	55.9
19.3	55.1	54.4	53.6	52.9	52.1	51.4	50.6
20.0	49.9	49.1	48.4	47.6	46.9	46.2	45.5
20.7	44.8	44.2	43.7	43.1	42.7	42.2	41.8
21.4	41.5	41.2	40.9	40.6	40.4	40.1	39.9
22.1	39.7	39.5	39.3	39.1	39.0	38.8	38.6
22.8	38.5	38.3	38.1	38	37.8	37.7	37.5
23.5	37.4	37.2	37.1	37.0	36.8	36.6	36.3
24.2	35.8	35.1	33.9	32.4	30.3	27.9	25.3
25.0	22.6	19.9	17.3	14.9	12.7	10.7	9.0
25.7	7.7	6.5	5.5	4.7	4.0	3.4	2.9
26.4	2.4	2.1	1.7	1.5	1.2	1.0	0.9
27.1	0.7	0.6	0.5	0.0	0.0	0.0	0.0

Step 3: Compute routing distance step, Δx, using equation 17–21. The factor 3,600 converts hours to seconds. A routing time interval of 0.1 hour is used because that is the time interval of the inflow hydrograph.

$$\Delta x = 0.5 \left((c_r 3,600 \Delta t + \frac{Q_{pi}}{T_r S_r c_r}) \right)$$

$$= 0.5 \left[4.297(3,600)(0.1) + \frac{684.18}{(44.6)(0.00091)(4.297)} \right]$$

$$= 2,735 \text{ ft}$$

This is longer than the reach length of 2,600 feet, so the routing is accomplished by treating the reach as a single routing distance step. Δx for the routing is 2,600 feet.

Muskingum X:

$$X = 0.5 \left(1.0 - \frac{Q_{pi}}{(T_r)(S_r)(c_r)(\Delta x)} \right)$$

$$= 0.5 \left(1.0 - \frac{684.18}{(44.6)(0.00091)(4.297)(2,600)} \right)$$

$$= -0.254$$

Step 4: Compute the Muskingum routing coefficients using equations 17–20 and 17–22.

Muskingum K:

$$K = \frac{\Delta x}{c_r}$$

$$= \frac{2,600}{4.297}$$

$$= 605.07 \text{ s}$$

Compute C_0, C_1, C_2, and C_3 using equations 17–11, 17–8, 17–9, and 17–10.

$$C_0 = \left(\frac{(0.1)3,600}{605.07} \right) + 2(1.0 - (-.254))$$

$$= 3.103$$

$$C_1 = \frac{\left[\frac{(0.1)3,600}{605.07} + 2(-.254) \right]}{3.103}$$

$$= 0.028$$

$$C_2 = \frac{\left[\frac{(0.1)3,600}{605.07} - 2(-.254) \right]}{3.103}$$

$$= 0.355$$

$$C_3 = \frac{\left[2(1.0 - (-.254)) - \frac{(0.1)3,600}{605.07} \right]}{3.103}$$

$$= 0.617$$

Check that $C_1 + C_2 + C_3 = 1.0$:

$$0.028 + 0.355 + 0.617 = 1.00$$

Step 5: Compute the Courant number and the grid Reynolds number. The grid Reynolds number D_g is the mathematical criterion that distinguishes laminar from turbulent flow. The Courant number C_g is the ratio of the physical wave celerity to the grid celerity $\Delta x / \Delta t$ (Ponce, 1989). These values are computed to determine if the routing is within accuracy limits of the Muskingum-Cunge method (app. 17–B).

These coefficients are nondimensional and defined as:

$$C_g = c_r (3,600) \frac{\Delta t}{\Delta x} \qquad \text{(eq. 17–32)}$$

Since K is defined in equation 17–22, a simpler equation for the Courant number is

$$C_g = \Delta t \times \frac{3,600}{K} \qquad \text{(eq. 17–33)}$$

$$C_g = \frac{0.1(3,600)}{605.07}$$

$$= 0.595$$

The grid Reynolds number is expressed as:

$$D_g = \frac{Q_{pi}}{T_r S_r c_r \Delta x} \qquad \text{(eq. 17–34)}$$

Since X is defined in equation 17–20 as:

$$X = \frac{1}{2}\left(1 - \frac{Q}{\left[T(S_o)(c)\Delta x\right]}\right)$$

A simpler equation for Grid Reynolds number (D_g) is:

$$D_g = 1 - 2X \qquad \text{(eq. 17–35)}$$

$$D_g = 1 - 2(-0.254)$$
$$= 1.508$$

The critical Grid Reynolds number is:

$$D_g(\text{critical}) = e^{(2.3C_g)} \qquad \text{(eq. 17–36)}$$

If D_g is less than D_g (critical), the routing conditions are within the accuracy limits. If D_g is greater than D_g (critical), the routing conditions are outside the accuracy limits.

For this example, D_g(critical) = $e^{2.3\,(0.595)}$ = 3.93.

In this case, the critical value of D_g is 3.93 and greater than the value of grid Reynolds number of 1.508, so the routing conditions are within the accuracy limits.

Step 6:   Solve the routing equation to determine the outflow hydrograph. Table 17–18 shows computations for the first five outflow discharges. The table starts at 10.0 hours when the hydrograph begins to rise, and ends at 13.6 hours after both the inflow and outflow hydrographs have peaked.

Plots of the inflow (upstream) and outflow (downstream) hydrographs for the reach, from 11.5 to 15.5 hours are shown in figure 17–16.

The channel, reach, and inflow hydrograph were entered to HEC–RAS, and an unsteady flow routing was completed using a normal flow downstream boundary condition. The peak outflow from HEC–RAS was 647 cubic feet per second, and the time of the peak outflow was 13.1 hours. Both are reasonable comparisons to the Muskingum-Cunge routing.

Table 17–18 Hydrograph routing for example 17–4

time hr	Inflow	Outflow	Notes
10.0	0.00	0.00	Start of hydrograph
10.1	0.74	0.26	$O_2 = C_1I_1 + C_2I_2 + C_3O_1$
			$0.26 = 0.028 \times 0.0 + 0.355 \times 0.74 + 0.617 \times 0.0$
10.2	1.13	0.58	$0.58 = 0.028 \times 0.74 + 0.355 \times 1.13 + 0.617 \times 0.26$
10.3	1.63	0.97	$0.97 = 0.028 \times 1.13 + 0.355 \times 1.63 + 0.617 \times 0.58$
10.4	2.27	1.45	$1.45 = 0.028 \times 1.63 + 0.355 \times 2.27 + 0.617 \times 0.97$
10.5	3.06	2.04	$2.04 = 0.028 \times 2.27 + 0.355 \times 3.06 + 0.617 \times 1.45$
10.6	4.01	2.77	
10.7	5.15	3.65	
10.8	6.48	4.70	
10.9	8.03	5.93	
11	9.82	7.37	
11.1	11.89	9.04	
11.2	14.29	10.99	
11.3	17.09	13.25	
11.4	20.37	15.88	
11.5	24.27	18.99	
11.6	29.31	22.80	
11.7	37.03	28.03	
11.8	50.70	36.33	
11.9	76.68	51.06	
12	119.72	76.15	
12.1	179.04	113.90	
12.2	254.20	165.53	
12.3	344.67	231.61	
12.4	442.98	309.81	
12.5	533.83	393.07	
12.6	604.92	472.22	
12.7	652.85	540.06	
12.8	678.15	592.24	
12.9	684.18	627.28	Peak of inflow hydrograph
13.0	672.05	644.77	
13.1	645.30	645.72	Peak of outflow hydrograph
13.2	608.62	632.54	
13.3	562.41	606.97	
13.4	509.94	571.28	
13.5	459.53	529.89	
13.6	416.94	487.82	

Figure 17–16 Plot of inflow and outflow hydrographs for example 17–4
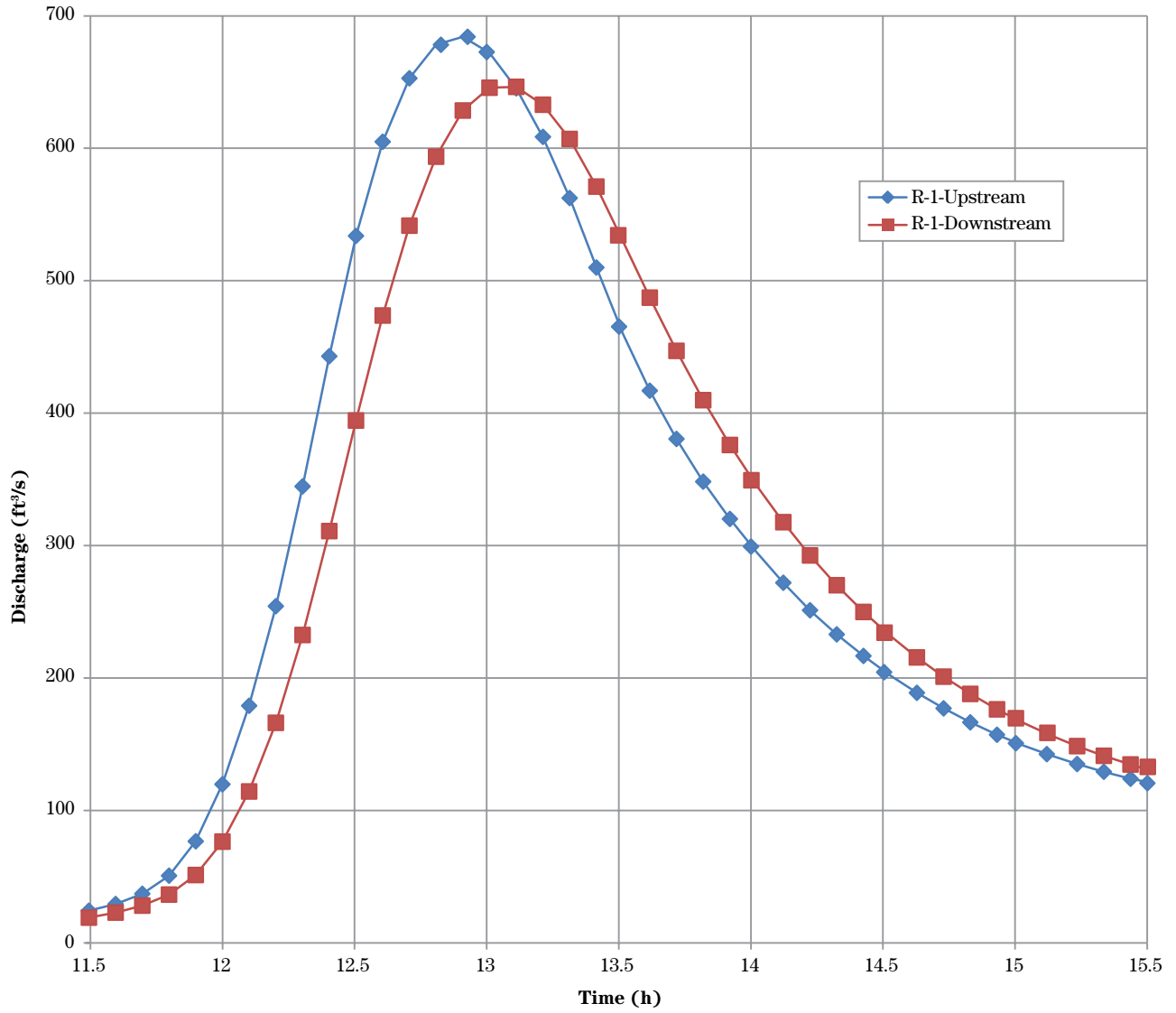
Example 17–5　Muskingum-Cunge routing of a floodplain cross section

This example shows the procedure to route a hydrograph through a floodplain reach. The cross section and reach properties are: channel bottom width is 15 feet, channel depth is 8 feet, channel side slope is 2:1, reach slope is 0.00091 foot per foot, Manning's n is 0.04, and reach length is 2,600 feet. At a depth of 8.73, the cross section top width is 655 feet, and at a depth of 13.73 feet, the cross section top width is 855 feet. Manning's n for the floodplain is 0.07.

The cross section and reach data were entered into HEC–RAS, and a relation of elevation, discharge, area, top width, and slope were calculated. Table 17–16 includes the complete cross section rating table used in this example. The table also includes the value of m (weighted log-log slope of the discharge versus area curve) at each elevation.

The inflow hydrograph (table 17–19) has been developed for the watershed upstream of the reach.

First routing distance step:

Step 1:　Find the number of time intervals to the peak discharge in table 17–19. The peak discharge of the inflow hydrograph is 3,385.4 cubic feet per second at time 12.8 hours. Five percent of the peak is 169 cubic feet per second. The closest inflow hydrograph coordinate to this value is 168 cubic feet per second at 10.3 hours, so this value is used in this example. At a time interval of 0.1 hour, there are 25 intervals to the peak. Since this is more than 10, the inflow hydrograph does not need to be interpolated at a smaller time increment.

Step 2:　Compute parameters for reference peak discharge of the inflow hydrograph. The reference discharge is the peak or 3,385.4 cubic feet per second. The routing coefficients are calculated based on hydraulic parameters of the peak discharge. Interpolate the reference end-area, reference top width, reference friction slope, and m from table 17–16 at the reference discharge of 3,385.4 cubic feet per second. The reference end-area is 1,956 square feet, the reference top width is 738.3 feet, the reference friction slope is 0.00091, and m is 1.0276.

Step 3:　Compute the reference wave celerity

$$c_r = \frac{(1.0276)3,385.4}{1,956}$$
$$= 1.779 \text{ ft/s}$$

Step 4:　Compute the routing distance step, Δx.

$$\Delta x = 0.5\left[c_r(3,600)\Delta t + \frac{Q_{pi}}{T_r S_r c_r} \right]$$
$$= 0.5\left[1.779(3,600)0.1 + \frac{3,385.4}{738.3(0.00091)1.779} \right]$$
$$= 1,736 \text{ ft}$$

This is shorter than the reach length of 2,600 feet so the routing is accomplished by treating the reach as two routing distance steps each with a 1,300 foot distance. Δx for the routing is 1,300 feet.

Step 5:　Compute Muskingum routing coefficients X and K, and C_0, C_1, C_2, and C_3.

Muskingum X

$$X = 0.5\left(1.0 - \frac{A_r}{T_r(S_r)m(\Delta x)} \right)$$
$$= 0.5\left[1.0 - \frac{1,956}{738.3(0.00091)1.0276(1,300)} \right]$$
$$= -0.590$$

Muskingum K

$$K = \frac{\Delta x}{c_r}$$
$$= \frac{1,300}{1.779}$$
$$= 730.75 \text{ s}$$

Compute C_0, C_1, C_2, and C_3.

$$C_0 = \left(\frac{0.1(3,600)}{730.75} \right) + 2(1.0 - (-0.590))$$
$$= 3.673$$

Table 17–19   Inflow hydrograph for example 17–5

Time hours	Discharge ft³/s	Discharge ft³/s	Discharge ft³/s	Discharge ft³/s	Discharge ft³/s	Discharge ft³/s	Discharge ft³/s
5.2	0.8	1.3	1.9	2.7	3.6	4.7	6.0
5.9	7.4	8.9	10.6	12.4	14.3	16.2	18.3
6.6	20.4	22.5	24.8	27.1	29.4	31.8	34.3
7.3	36.7	39.3	41.8	44.5	47.1	49.8	52.5
8.0	55.2	58.0	60.8	63.7	66.7	69.8	73.1
8.7	76.6	80.4	84.5	88.9	93.8	98.9	104.4
9.4	110.2	116.2	122.4	128.5	134.7	141.0	147.3
10.1	153.8	160.7	168.0	176.0	184.8	194.5	205.3
10.8	217.4	230.8	245.8	262.6	281.4	302.6	326.7
11.5	354.6	389.8	441.5	527.0	674.4	901.8	1,204.1
12.2	1,576.3	2,008.4	2,458.0	2,853.7	3,144.5	3,319.5	3,385.4
12.9	3,360.4	3,252.5	3,080.4	2,865.2	2,612.2	2,340.5	2,087.7
13.6	1,875.9	1,695.6	1,537.5	1,399.9	1,277.9	1,166.9	1,068.2
14.3	982.2	905.8	837.4	776.6	722.7	674.6	632.2
15.0	595.1	562.2	533.1	507.1	483.6	462.5	443.3
15.7	426.0	410.0	395.2	381.8	369.3	357.2	345.5
16.4	334.2	323.2	312.7	303.0	294.3	286.9	280.6
17.1	274.7	269.4	264.4	259.9	255.6	251.5	247.7
17.8	244.0	240.5	237.1	233.8	230.5	227.4	224.3
18.5	221.2	218.2	215.3	212.3	209.4	206.5	203.6
19.2	200.8	197.9	195.1	192.2	189.4	186.6	183.7
19.9	180.9	178.1	175.3	172.5	169.8	167.1	164.5
20.6	162.1	159.8	157.7	155.7	153.9	152.3	150.8
21.3	149.5	148.2	147.1	146.1	145.2	144.3	143.4
22.0	142.6	141.9	141.1	140.4	139.7	139.1	138.4
22.7	137.8	137.2	136.6	136.0	135.4	134.8	134.2
23.4	133.6	133.1	132.5	131.9	131.4	130.7	129.6
24.1	127.7	124.9	120.9	115.2	107.9	99.4	90.1
24.8	80.5	70.9	61.7	53.1	45.2	38.1	32.1
25.5	27.2	23.1	19.6	16.7	14.2	12	10.2
26.2	8.6	7.3	6.2	5.2	4.4	3.7	3.1
26.9	2.6	2.2	1.9	1.6	1.3	1.1	0.9

$$C_1 = \frac{\left[\dfrac{0.1(3,600)}{730.75} + 2(-0.590)\right]}{3.673}$$

$$= -0.187$$

$$C_2 = \frac{\dfrac{0.1(3,600)}{730.75} - 2(-0.590)}{3.673}$$

$$= 0.455$$

$$C_3 = \frac{\left[2\left(1.0 - (-0.590)\right) - \dfrac{0.1(3600)}{730.75}\right]}{3.673}$$

$$= 0.732$$

Check that $C_1 + C_2 + C_3 = 1.0$:

$-0.187 + 0.455 + 0.732 = 1.000$

Step 6: Compute Courant number C_g and grid Reynolds number D_g.

$$C_g = \frac{\Delta t(3,600)}{K}$$

$$= \frac{0.1(3,600)}{730.75}$$

$$= 0.493$$

$$D_g = 1 - 2(X)$$

$$= 1 - 2(-0.590)$$

$$= 2.18$$

The equation 17–35 defining D_g (critical) is repeated for convenience:

$$D_g(critical) = e^{\left(\frac{2.3}{C_g}\right)}$$

If D_g is less than D_g (critical), the routing conditions are within the accuracy limits. If D_g is greater than D_g (critical), the routing conditions are outside the accuracy limits.

In this case the critical value of D_g is 3.1, so the routing conditions are within the accuracy limits.

Step 7: Solve the routing equation to determine the outflow hydrograph. Table 17–20 shows com-

putations for the first five outflow discharges. This will complete the routing for the first routing distance step of 1,300 feet. Steps 1 through 5 are repeated for routing through the second routing distance step.

Second routing distance step:

Step 1: Compute parameters for inflow peak discharge for routing distance step 2. The outflow from routing distance step 1 is the inflow to routing segment 2. The reference discharge is the peak or 3,072.61 cubic feet per second (table 17–20). The routing coefficients are calculated based on hydraulic parameters of the peak inflow discharge. Interpolate the reference end-area, reference top width, reference friction slope and *m* from table 17–16 at the reference discharge of 3,072.61 cubic feet per second. The reference end-area = 1,804.5 square feet, the reference top width = 730.1 feet, the reference friction slope = 0.00091, and *m* = 1.011.

Compute the reference wave celerity.

$$c_r = \frac{1.011(3,072.61)}{1,804.5}$$

$$= 1.72 \text{ ft/s}$$

Step 2: Compute the routing distance step, Δx. This calculation is not needed for routing distance step 2. The routing distance step was determined during routing distance step 1 as 1,300 feet.

Step 3: Compute the Muskingum routing coefficients.

Muskingum X:

$$X = 0.5\left(1.0 - \frac{A_r}{T_r(S_r)m(\Delta x)}\right)$$

$$= 0.5\left[1.0 - \frac{1,805.5}{730.1(0.00091)1.011(1,300)}\right]$$

$$= -0.533$$

Muskingum K:

$$K = \frac{\Delta x}{c_r}$$

$$= \frac{1,300}{1.72}$$

$$= 755.81 \text{ s}$$

Table 17–20 Hydrograph routing for example 17–5

Time (hr)	Inflow	Outflow for routing distance step 1	Notes	Outflow for routing distance step 2	Notes
5.1	0	0.00	Start of hydrograph	0	Start of hydrograph
5.2	0.8	0.36	$O_2=C_1I_1+C_2I_2+C_3O_1$	0.16	$O_2=C_1I_1+C_2I_2+C_3O_1$
			$0.36 = -.187 \times 0 + 0.455 \times 0.8 + 0.732 \times 0$		$0.16 = -.166 \times 0 + 0.435 \times 0.36 + 0.731 \times 0$
5.3	1.27	0.69	$0.69 = -.187 \times 0.8 + 0.455 \times 1.27 + 0.732 \times 0.36$	0.36	$0.36 = -.166 \times 0.36 + 0.435 \times 0.69 + 0.731 \times 0.16$
5.4	1.89	1.13	$1.13 = -.187 \times 1.27 + 0.455 \times 1.89 + 0.732 \times 0.69$	0.64	$0.64 = -.166 \times 0.69 + 0.435 \times 1.13 + 0.731 \times 0.36$
5.5	2.67	1.69	$1.69 = -.187 \times 1.89 + 0.455 \times 2.67 + 0.732 \times 1.13$	1.01	$1.01 = -.166 \times 1.13 + 0.435 \times 1.69 + 0.731 \times 0.64$
5.6	3.62	2.38	$2.38 = -.187 \times 2.67 + 0.455 \times 3.62 + 0.732 \times 1.69$	1.50	$1.50 = -.166 \times 1.69 + 0.435 \times 2.38 + 0.731 \times 1.01$
5.7	4.73	3.22		2.10	
5.8	5.99	4.20		2.83	
5.9	7.4	5.32		3.68	
6	8.95	6.58		4.67	
6.1	10.62	7.98		5.79	
6.2	12.39	9.49		7.04	
6.3	14.26	11.12		8.41	
6.4	16.22	12.85		9.89	
6.5	18.25	14.68		11.48	
6.6	20.36	16.60		13.18	
6.7	22.53	18.59		14.96	
6.8	24.77	20.67		16.84	
6.9	27.06	22.81		18.80	
7	29.41	25.02		20.84	
7.1	31.81	27.29		22.95	
7.2	34.25	29.61		25.13	
7.3	36.74	31.99		27.37	
7.4	39.28	34.42		29.67	
7.5	41.85	36.89		32.02	
7.6	44.46	39.41		34.42	
7.7	47.1	41.96		36.88	
7.8	49.78	44.56		39.37	
7.9	52.5	47.20		41.92	
8	55.24	49.86		44.50	
8.1	58.02	52.57		47.12	

Table 17–20 Hydrograph routing for example 17–5—continued

Time (hr)	Inflow	Outflow for routing distance step 1	Notes	Outflow for routing distance step 2	Notes
8.2	60.84	55.31		49.78	
8.3	63.73	58.11		52.48	
8.4	66.7	60.97		55.24	
8.5	69.8	63.91		58.06	
8.6	73.07	66.98		60.97	
8.7	76.58	70.21		63.99	
8.8	80.36	73.64		67.15	
8.9	84.47	77.31		70.50	
9	88.93	81.26		74.05	
9.1	93.75	85.51		77.83	
9.2	98.94	90.08		81.89	
9.3	104.45	94.96		86.21	
9.4	110.23	100.13		90.82	
9.5	116.23	105.57		95.69	
9.6	122.35	111.21		100.80	
9.7	128.53	117.01		106.12	
9.8	134.73	122.92		111.62	
9.9	140.96	128.92		117.27	
10	147.28	135.02		123.06	
10.1	153.8	141.27		129.00	
10.2	160.66	147.75		135.12	
10.3	168.02	154.56		141.48	
10.4	176.02	161.81		148.15	
10.5	184.81	169.62		155.22	
10.6	194.52	178.11		162.79	
10.7	205.33	187.42		170.96	
10.8	217.4	197.71		179.86	
10.9	230.85	209.11		189.62	
11	245.84	221.76		200.37	
11.1	262.59	235.83		212.24	
11.2	281.38	251.55		225.43	
11.3	302.62	269.21		240.14	
11.4	326.75	289.14		256.63	

Table 17–20 Hydrograph routing for example 17–5—continued

Time (hr)	Inflow	Outflow for routing distance step 1	Notes	Outflow for routing distance step 2	Notes
11.5	354.61	311.90		275.27	
11.6	389.77	339.34		297.06	
11.7	441.54	376.41		324.56	
11.8	527.05	432.77		363.03	
11.9	674.44	525.10		421.95	
12	901.85	668.60		512.12	
12.1	1,204.06	868.61		641.22	
12.2	1,576.30	1,127.88		815.17	
12.3	2,008.36	1,444.65		1,037.08	
12.4	2,458.01	1,800.31		1,301.43	
12.5	2,853.72	2,156.62		1,590.63	
12.6	3,144.52	2,475.76		1,881.70	
12.7	3,319.52	2,734.61		2,154.11	
12.8	3,385.40	2,921.34	Peak upstream	2,391.49	
12.9	3,360.36	3,034.32		2,583.16	
13	3,252.47	3,072.61	Peak at routing distance step 1	2,721.18	
13.1	3,080.42	3,042.53		2,802.63	
13.2	2,865.23	2,954.77		2,828.99	Peak at routing distance step 2
13.3	2,612.25	2,815.67		2,802.31	
13.4	2,340.49	2,637.50		2,728.40	

Compute C_0, C_1, C_2, and C_3.

$$C_0 = \left(\frac{0.1(3,600)}{755.81} \right) + 2\left(1.0 - (-0.533)\right)$$
$$= 3.542$$

$$C_1 = \frac{\left(\frac{0.1(3,600)}{755.81} \right) + 2(-0.533)}{3.542}$$
$$= 0.166$$

$$C_2 = \frac{\frac{0.1(3,600)}{755.81} - 2(-0.533)}{3.542}$$
$$= 0.435$$

$$C_3 = \frac{\left[2\left(1.0 - (-0.533)\right) - \frac{0.1(3,600)}{755.81} \right]}{3.542}$$
$$= 0.731$$

Check that $C_1 + C_2 + C_3 = 1.0$:

$- 0.166 + 0.435 + 0.731 = 1.000$.

Step 4:   Compute Courant number C_g and grid Reynolds number D_g.

$$C_g = \frac{\Delta t(3,600)}{K}$$
$$= \frac{0.1(3,600)}{755.81}$$
$$= 0.476$$

$$D_g = 1 - 2(X)$$
$$= 1 - 2(-0.533)$$
$$= 2.066$$

The D_g (critical) for this example is calculated:

$$D_g(\text{critical}) = e^{2.3(0.476)}$$
$$= 2.99$$

This is greater than the D_g value of 2.066, so the routing conditions are within accuracy limits.

Step 5:   Solve the routing equation to determine the outflow hydrograph. Table 17–20 shows com-

putations for the first five outflow discharges. This will complete the routing at the end of the second routing distance step.

Plots of the upstream inflow, reach midpoint (routing distance step 1), and outflow (routing distance step 2) hydrographs for the reach are shown in figure 17–17.

The cross section, reach, and inflow hydrograph were entered to HEC–RAS, and an unsteady flow routing was completed using a normal flow downstream boundary condition. The peak outflow from HEC–RAS was 2,902 cubic feet per second, and the time of the peak outflow was 13.2 hours. Though the comparison with unsteady flow results are not as good as with the channel routing of example 17–4, nevertheless, the comparison is reasonable.
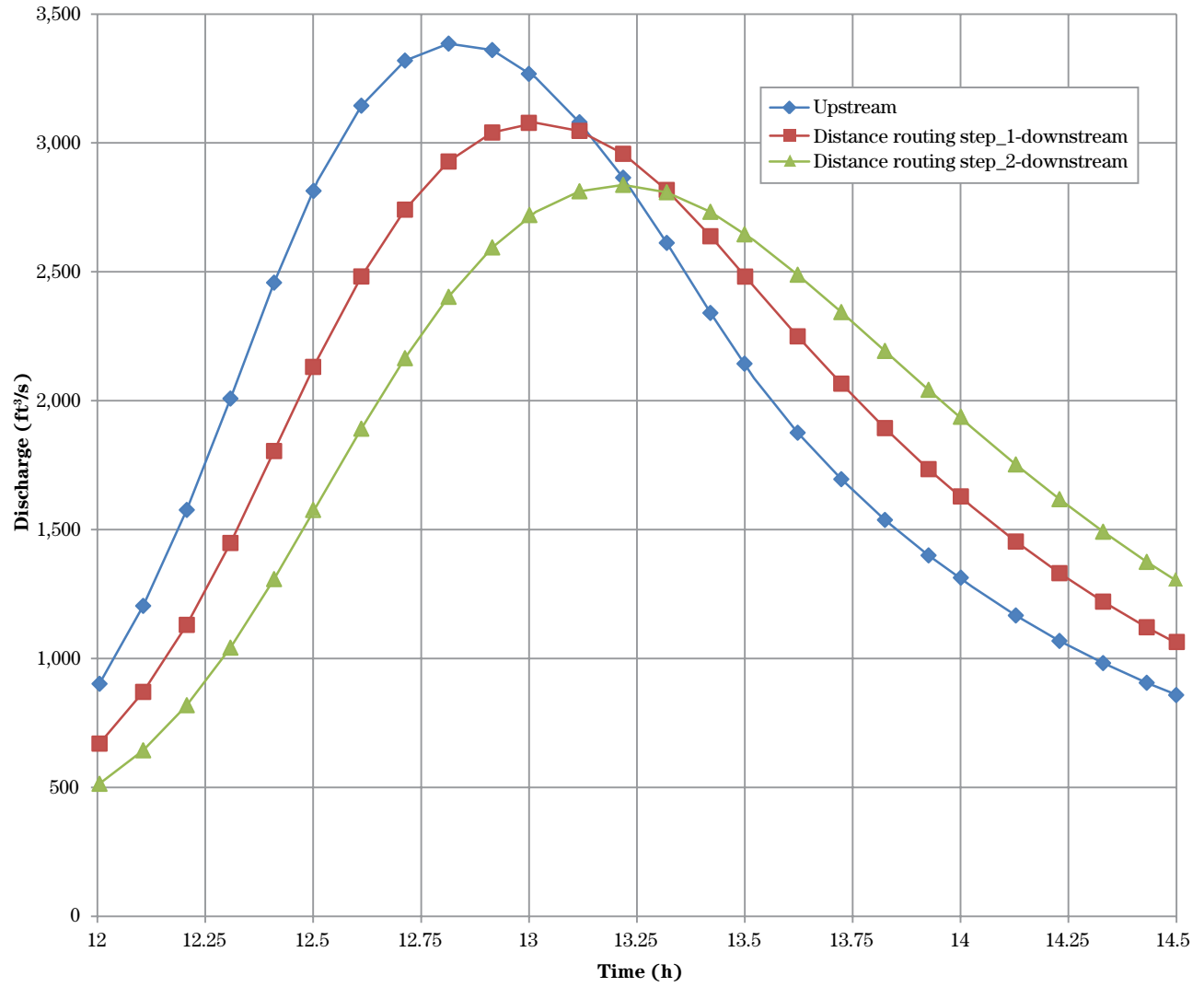
(i)   Effects of baseflow on routed flows

The amount of baseflow in a stream is based on several factors such as land use, geology, soils, size of drainage area, and climate. In many areas of the country in watersheds of the general size considered by NRCS, baseflow is negligible in comparison to peak flood flows and is not added to the hydrographs at reaches. If baseflow is significant, it should be included in the hydrograph before routing it through the reach. In most cases, baseflow is considered a constant value and is simply added to the discharge at each time step of the runoff hydrograph entering the upstream end of the reach.

WinTR–20 is an event-based rainfall-runoff model, so runoff is assumed to be a direct consequence of rainfall. A constant baseflow is an optional data entry by the user. The baseflow is added to the runoff hydrograph and then routed through the reach, either a structure reach or a channel reach. If the reach is a structure reach, the starting and ending elevations are set at elevations to pass the baseflow according to the elevation versus discharge table.

In general, if baseflow is added at a structure, the peak elevation during the routing will be slightly higher. If the reach is a channel reach, the baseflow is added to the hydrograph, and the routing coefficients are computed based on the peak discharge. If the peak discharge for the reach includes baseflow, the routing coefficients will be slightly different than if baseflow

Figure 17–17   Hydrograph plot for example 17–5

were not added. The hydrograph with baseflow is routed using these routing coefficients. The peak of the outflow hydrograph is generally higher than if baseflow were not considered.

(j) Effects of transmission losses on routed flows

A flood hydrograph is altered by transmission losses occurring during passage of the flow through a reach. The amount of loss depends on the percolation rate of the channel, the wetted perimeter of the channel during flow, and the duration of flow for a particular wetted perimeter (NEH630.19). Transmission loss varies with the amount of flow in the channel which means that the most accurate method of deducting the transmission loss from the routed flow will be on an incremental flow basis. It is seldom worthwhile to handle it in this manner unless the transmission loss is very large.

An acceptable practice for handling transmission loss is to route the inflow hydrograph in the usual manner and afterwards deduct a suitable quantity of flow from the outflow hydrograph. If that outflow is to be routed downstream again, the manner of flow deduction will not be critical. In some cases, it may be reasonable to assume that local inflow will be completely absorbed by transmission losses, thus no local inflow is added to the unmodified outflow hydrograph. In other cases, local rainfall may completely satisfy transmission losses, requiring unmodified local inflow to be added to the unmodified outflow hydrograph. The use of detailed procedures outlined in NEH630.19, Transmission Losses, may be necessary for complex situations.

## (k) Routing through a system of channels and reservoirs

The methods of routing given in examples 17–1, 17–2, 17–3, 17–4, and 17–5 are used for individual reservoirs and reaches of a stream system. Ordinarily, a routing progresses from reach through reach until stages, rates, or volumes of flow are known for selected points in the entire stream system of a watershed. The method of progression will be illustrated using a schematic diagram of a stream system. A sample schematic is given in figure 17–18. It does not need to be drawn to scale. The main purpose of the schematic is to show

the reaches and reservoirs in their proper relation to each other, but various kinds of data can be written down at their respective points to make the schematic a complete reference during the routing.

The schematic diagram for the reaches and reservoirs in a watershed can be drawn after the watershed has been divided into subareas and reaches are identified. In dividing the watershed into subareas and reaches, many factors need to be considered. These are described in NEH630.06, Stream Reaches and Hydrologic Units (NRCS 1998).

Three major considerations of routing flows through a watershed system are: at what locations in the watershed are peak discharges, hydrographs, or other hydrologic or hydraulic information desired; what kinds of alternative project or land use actions will be evaluated; and if there is a variation in land use and hydrologic soil group, at what locations should subareas be defined to make each one relatively homogenous. For example, if there are agricultural or urban flood damages that are being evaluated, subareas and reaches need to be defined at those specific locations. If there are particular locations where projects, such as flood water retarding structures, levees, etc., are being considered, subareas and reaches need to be defined at those locations in the watershed.

Figure 17–18   Schematic diagram for watershed
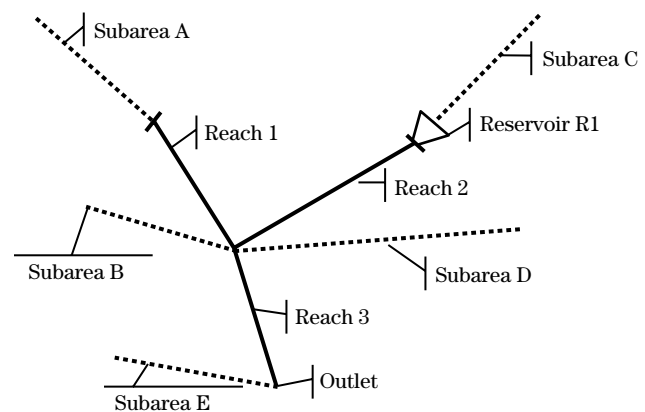
Locations of existing and proposed reservoirs should be shown on the schematic diagram. Elevation-discharge-storage relations for all existing and proposed reservoirs in the watershed are needed and should be developed based on procedures outlined earlier in this chapter.

The first major step in routing through reaches in the stream system is to develop the cross section rating tables for each reach. In a typical hydrologic analysis, water surface profiles are run for the main stream and major tributaries (NEH630.14). If there are reaches in upper parts of the watershed where water surface profiles have not been run, cross section data may be collected in several different ways depending on the purpose of the analysis, time and budget available, and judgment of the trade-off of detail versus quality of routing results. Cross sections may be defined from topographic maps, hand-level surveys, geographic information systems, or engineering surveys. The cross section rating table may be developed using HEC–RAS or may be derived from solution of Manning's equation at a range of discharges based on the estimated Manning's *n* and channel slope.

The flow conditions in each reach are represented by one cross section rating table. A single cross section within a reach may be selected as being representative. Alternatively, a composite rating table may be developed, which is a weighting of the individual cross section properties. The cross section properties used in the Muskingum-Cunge channel routing procedure are discharge, end-area, top width, and friction slope. In selecting a representative cross section or developing a composite, the discharge and end-area are of the greatest importance because they reflect the velocity and reach storage characteristics.

The following concepts for routing reaches and reservoirs in a watershed describe the way in which WinTR–20, SITES, and WinTR–55 computer programs operate. These rainfall-runoff event models develop hydrographs and route them on a storm-by-storm basis. Generally, a rainfall amount and distribution are assumed to occur uniformly over the watershed. Hydrographs for individual subareas are developed and then routed through the reach system. Routing through a stream system begins at the head of the uppermost reach. If there is more than one starting place, as in figure 17–18, the most convenient should be chosen.

The sequence of operations followed in routing the hydrographs from a single storm over the watershed is illustrated for the schematic diagram in figure 17–18.

Step 1: Develop a hydrograph for Subarea A using procedures of NEH630.16. The basic data needed are drainage area, runoff curve number, time of concentration, dimensionless unit hydrograph, antecedent runoff condition, rainfall amount, and rainfall distribution.

Step 2: Route the hydrograph from Subarea A through Reach 1 using the Muskingum-Cunge routing procedure using the cross section rating table and reach length representative of Reach 1.

Step 3: Develop a hydrograph for Subarea B.

Step 4: Add the hydrograph from the downstream end of Reach 1 and the hydrograph from Subarea B.

Step 5: Develop a hydrograph for Subarea C.

Step 6: Route the hydrograph from Subarea C through Reservoir R1 using the Storage-Indication routing procedure.

Step 7: Route the outflow hydrograph from Reservoir R1 through Reach 2.

Step 8: Develop a hydrograph for Subarea D.

Step 9: Add the hydrograph from the downstream end of Reach 2 and the hydrograph from Subarea D.

Step 10: Add the hydrograph from the downstream end of Reach 1 and the hydrograph from the downstream end of Reach 2.

Step 11: Route the hydrograph from step 10 (the result of the addition of hydrographs from the two branches) through Reach 3.

Step 12: Develop a hydrograph for Subarea E.

Step 13: Add the hydrograph from the downstream end of Reach 3 and the hydrograph from Subarea E.

Step 14: Proceed with developing, routing, and adding hydrographs in a similar manner until the watershed Outlet is encountered.

The user manuals for WinTR–20, SITES, and WinTR–55 include information on how to organize and enter data

related to subareas, reaches, reservoirs, and watershed schematics.

(l)   Routing through tide gates and dikes

In coastal or riverine areas, dikes are often constructed to protect urban or agricultural land from damage from high tides, wave action, storm surge, or floodwater. Pipe outlets or gates are generally constructed to control the flow of water, which will allow water from the urban or agricultural land to drain when the water level is higher inside the diked area. Conversely, flap gates or mechanical gates are provided for the pipe or gate outlet to prevent the flow of water from damaging the land when the water level is higher outside the diked area.

Various criteria and standards have been published to design dike embankments with respect to location, height, top width, side slope, and foundation. This description is restricted to pipe and gate structures, which control the flow of water. To some extent, the design of pipes or gates may control the height of the dike. However, in most cases, the height of the dike is controlled by other considerations such as high tide, storm surge, or river stage. If the design height of the dike is already determined, the pipe or gate structures are generally designed to drain the land behind the dike of a certain quantity of water in a certain period of time. A serious situation can develop if the water behind the dike is high and cannot be drained. Scenarios of various situations can be evaluated through engineering analyses to help define the risk of flooding related to the water level behind the dike and the associated construction and maintenance costs. By analyzing several sizes and locations of pipes or gates, an appropriate benefit-cost ratio can be estimated.

Two methods of routing through tide gates are described. The first method is a hand calculation method published in Technical Release 1 (USDA–SCS 1955). The second is use of the HEC–RAS computer program (U.S. Army Corps of Engineers 2010).

Technical Release 1—Technical Release 1 includes hand computation steps and an example to route flow through tide gates. It is based on sound engineering principles; however, it includes several simplifying assumptions. These are:

- The tide elevation versus time curve is cyclic.
- The inflow rate from the land side of the dike is constant.
- The outlet pipe is submerged at all times.
- No reverse flow occurs through the outlet.
- The tide gate opens or closes at the instant the head on the pipe varies from zero.

The complete example in the technical release shows computations to analyze a series of gate opening and closing times for a range of pipe sizes and number of pipes to select a project such that the benefits exceed the costs. Even though the example includes many trial-and-error calculations, it would be possible to use the procedure to analyze one pipe size and one gate closing time/elevation with much less effort. The principle upon which the technical release is based is that during the time the gate is closed (water outside the dike is higher than water inside the dike), the inflow is stored, and a maximum water level is reached. After the gate opens (water inside the dike is higher than water outside the dike), the pipe allows the water to drain down to the starting minimum water level. The calculations represent a cycle and the same minimum and maximum water levels are assumed for each day. However, when designing such a system, a level of protection concept is recommended when setting the tide elevation versus time curve and the inflow rate, both of which would vary throughout the year for various tide surge and storm events.

Use of HEC–RAS to analyze flow at tide gates—HEC–RAS is a versatile computer program that may be used to analyze most situations found in streams and rivers with respect to water flowing at a constant rate (steady flow) or at a variable rate (unsteady flow). A particular characteristic of routing through tide gates is that the water level outside the diked area varies with time. In the case of tides, this variation is generally a high tide-low tide cycle within a 24-hour period.

HEC–RAS may be used to analyze a single pipe or gate outlet for a diked area or multiple pipes or gates from several connected or disconnected diked areas. The complexity of analysis could be determined by the project scope and purpose, the expertise of the professional staff, and the time available to complete the analyses. For example, a complex system of dikes and gates may be simplified into a single typical case

or several separate HEC–RAS analyses. Some of the capabilities of HEC–RAS along with data requirements to analyze a single pipe or gate will be described next. Using the unsteady flow option in HEC–RAS provides the most powerful way to obtain a good technical analysis.

The first step in utilizing HEC–RAS with unsteady flow is to gather geometric data including cross section data, roughness values, dike elevations, pipe and/or gate characteristics. The drainage area to the pipe or gate is necessary. A hydrograph from this drainage area may be entered into HEC–RAS to represent the flow to be routed. The hydrograph is not developed by HEC–RAS, but must be estimated by another procedure such as WinTR–20 or WinTR–55.

When using the HEC–RAS unsteady flow option, an initial flow is required. These initial conditions could represent baseflow, or if the system has no baseflow, a small channel (often called a pilot channel) is placed in each cross section to carry an initial flow so the computations may be completed. A "warm up" period of several hours of constant discharge and elevation is recommended at the start of the HEC–RAS unsteady flow computations to ensure the hydraulics are at an equilibrium before the actual tidal or flood routing. Sometimes the computations are unstable for the initial discharges selected. In this case, increase the initial discharge or make another adjustment such as reduce the size of the pilot channel.

The variation of water level outside the diked area is treated as a downstream boundary condition within HEC–RAS. It may be represented as a time series of elevation versus time. Multi-day analyses are possible with HEC–RAS. Forty-eight or 72 hour simulation periods may be run with an efficient execution time.

Gate structures and culverts may be analyzed with respect to flow and backwater effects. The gate (or gates) may be entered as an element of an in-line structure or a lateral weir in HEC–RAS. The HEC–RAS computer program has two practical options for analyzing flow through the gates. One of these is the time series of gate opening and closing. With this option, the user can specify the initial gate opening height and opening height at each time step throughout the simulation. The gate can actually be completely closed if desired allowing no flow through the gate. The other option for gate opening and closing is to have the gate

operation depend on elevation of water upstream and downstream of the gate. For example, when the tail-water rises to be equal to the headwater, the gate can be automatically closed and, with falling tide, when the headwater becomes greater than the tailwater, the gate can be opened.

In HEC–RAS version 4.1, culverts, which are elements of an in-line weir, cannot have flap gates to prevent backflow. However, if the culvert is an element of a lateral weir, flow can be prevented in either direction of flow. Setting up HEC–RAS to do unsteady flow for lateral weirs is more complicated than in-line weirs. So if an in-line weir is the approach to be used, the culvert could be treated as an equivalent gate structure, which can be opened or closed based on headwater and tailwater elevations.

The unsteady flow computation procedure in HEC–RAS is sensitive to various factors and can produce unstable results or just quit computing at a particular time step. However, HEC–RAS most often gives clues as to what caused the computational problem. Viewing the various graphic and tabular output can generally lead to the necessary data modifications. The problem encountered most often is the instability caused by the routing distance step or time step being too large. In these cases, the time step may be changed in the Unsteady Flow Analysis window. The routing distance step may be changed by deleting interpolated cross sections and interpolating new cross sections at a shorter routing distance step. Analyzing gates and pipes with HEC–RAS unsteady flow gives a very clear picture of the operation during complex hydraulic and hydrologic conditions. For complex and expensive projects, it is well-worth the effort it takes to do multiple simulations to get HEC–RAS to model the actual situation.

630.1704   References

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Appendix A   Comparisons of the Muskingum-Cunge Method with Dynamic Wave Unsteady Flow Routings

Background and introduction

With the arrival of sophisticated mathematical solution techniques and increased computer speed and storage, comparison of simplified routing techniques to the unsteady flow equations (referred to as "dynamic wave" or "St. Venant" equations) has become much easier. Comparisons of mathematical routing techniques to hypothetical situations or to actual measured hydrographs are now possible. For example, a hypothetical hydrograph can be routed through a hypothetical channel reach, a hypothetical hydrograph may be routed through an actual channel reach, and measured hydrographs may be routed through an actual channel reach. All of these types of comparisons have been reported in the professional literature. There are one- and two-dimensional formulations and solutions of the dynamic wave equations. In this appendix, only the one-dimensional formulation and solution are described.

There are several purposes for conducting these comparisons. One is to compare various simplified routing techniques to the dynamic wave solution (which represents a "correct" solution) to see which one has the best performance. Another purpose is to determine limits of simplified routing techniques. By determining limits of application, engineers may select a routing model that will provide the most realistic solution to the situation being analyzed. Two types of routing applications are clearly suited to the dynamic wave equations instead of simplified routing techniques. One of these is in areas influenced by tides where the backwater changes with time. The other is dam breach routing where the friction slope changes rapidly with time and distance down the reach. Between these two extremes, there are situations where simplified techniques provide reasonably accurate results. The question is at what point do the simplified routing procedures diverge from the "correct" solution.

Even though the concept is simple and straightforward, the actual details involved in organizing test data to be analyzed by simplified and dynamic routing techniques can be daunting. There are an infinite number of combinations of data that can be tested. For example, typical data needed to conduct a routing comparison include an inflow hydrograph, channel and/or valley cross section data, and reach length. An inflow hydrograph can have a particular peak discharge, time of rise to the peak discharge, and volume of water.

Channels and valleys can have a particular depth, width, and roughness. Reach lengths can have a particular length and slope. To add even more complexity, upstream and downstream boundary conditions must be established and, if necessary, lateral inflow to the reach, tributary inflows, changing channel/valley cross sections, and changing roughness could be analyzed.

Two approaches were combined to attempt to organize a comprehensive test of simplified routing procedures. These are explained in Younkin and Merkel (1986). One approach involved using a nondimensional analysis of important parameters, which control the variability of solutions of the dynamic wave equations. The first major nondimensional factor is linked to Froude number, which is a dimensionless number defined as the ratio of a characteristic velocity to a gravitational wave velocity. It may equivalently be defined as the ratio of a body's inertia to gravitational forces.

Low Froude numbers are associated more with storage-type routings, and higher Froude numbers are associated more with kinematic-type routings. The other three nondimensional factors are related to cross section width/depth ratio, reach length, and rise time of the inflow hydrograph. The rise time of the inflow hydrograph is associated with the unsteadiness of the inflow hydrograph. A short rise time is highly unsteady, and a long rise time is closer to a steady flow situation. Each of these nondimensional factors was represented by a single number, which may be computed for a set of specific test conditions. The second approach or aspect of the analysis was to estimate the range of these nondimensional factors, that occur in NRCS hydrologic and hydraulic projects.

Typical NRCS projects are conducted for watersheds less than 400 square miles, many less than 10 square miles. Hydrologic and hydraulic data for numerous completed projects around the United States were analyzed to determine the range of nondimensional factors. The reason for this was to study the range of nondimensional factors that actually occur in reality; thus limiting the time needed to conduct the study and making the results and conclusions more practical. Limiting the scope of testing was further achieved by considering how the nondimensional factors related to each other for a given watershed. For example, a low value of hydrograph rise time (fast rise time) was

not associated with a low value of Froude number or a long value of nondimensional reach length.

Basic assumptions used in the tests were to assume a prismatic cross section (cross section shape and roughness were constant throughout the reach) and to assume three downstream boundary conditions (normal depth, fixed stage, and critical depth).

A total of 216 tests were run, and results were as expected; for longer hydrograph rise times, the simplified routing models were more accurate than with the short hydrograph rise times. Cross section shape had less sensitivity than the hydrograph rise time.

More recently, Merkel (2002) completed a comparison of the Muskingum-Cunge and dynamic wave model FLDWAV (Fread and Lewis 1988). Test conditions are described in Merkel (2002) and consisted of a combination of actual cross sections and hypothetical cross sections (such as trapezoidal channels with and without floodplain). A range of inflow hydrographs and reach lengths were tested. In addition to evaluating the peak discharge at the end of the reach, the travel time of the peak discharge was also evaluated. The comparison plots in the three figures (figs. 17A–1, 17A–2, and 17A–3) are based on the value of Q*, which represents attenuation for the reach.

$$Q^* = \frac{Q_{po} - Q_b}{\left(Q_{pi} - Q_b\right)} \qquad \text{(eq. 17A–1)}$$

where:
Q^* = discharge ratio calculated from peak discharges resulting from Muskingum-Cunge routings or from the dynamic wave model (FLDWAV), dimensionless
Q_{pi} = peak discharge of inflow hydrograph, ft³/s
Q_{po} = peak discharge of outflow hydrograph, ft³/s
Q_b = baseflow, ft³/s

Discussion

The channel tests had a correlation coefficient of 0.97 but slightly underestimated the peak discharge (the regression line is slightly below a 1:1 relation). The floodplain tests had a correlation coefficient of 0.70 and slightly overestimated the peak discharge (the regression line is slightly above a 1:1 relation). The plot of all 77 tests showed a correlation coefficient of 0.86 and a regression line on the 1:1 relation; meaning the total of all results are not biased on the high or low side.

The error in outflow peak time was computed based on the travel time of the peak discharge. The error is the time to peak outflow (Muskingum-Cunge) minus the time to the peak outflow (dynamic wave) divided by the time to peak outflow minus the time to the peak inflow of the dynamic wave routing. For the 77 tests, the average error in this timing for the Muskingum-Cunge was 6.8 percent with most values (60) ranging from plus to minus 20 percent. Errors in timing seemed to be random and not specifically related to any cross section or flow conditions. The minimum travel time in any routing distance step (and this included the solution of the dynamic wave equations) was one time step. Of the 77 tests run, 48 had a timing error of one time step or less; 19 had a timing error of two time steps. This left 10 tests with a timing error of more than two time steps. The conclusion is that timing of routings is adequate.

Since the floodplain tests had more variability, there seems to be a problem modeling the interaction of the channel and floodplain or representing how the cross section stage-discharge-area relation is used to calculate the Muskingum-Cunge routing coefficients. This points to the need for further testing and development. These results also point to the need to identify under what conditions the Muskingum-Cunge procedure produces a significantly higher or lower peak discharge than the dynamic wave model.

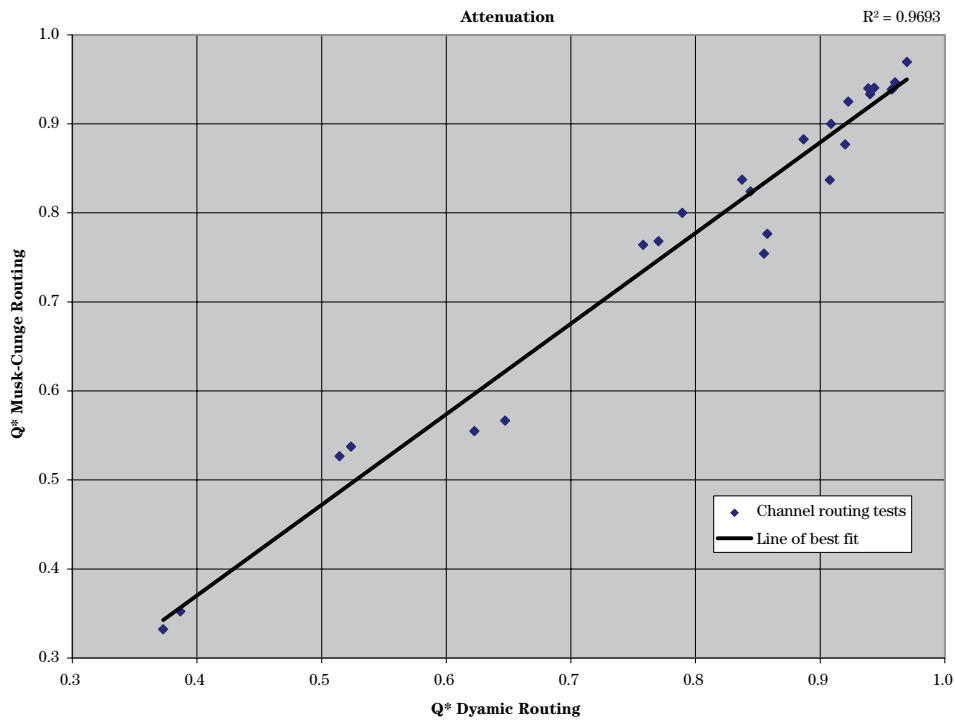**Figure 17A–1**   Comparison of peak discharges for 25 channel tests

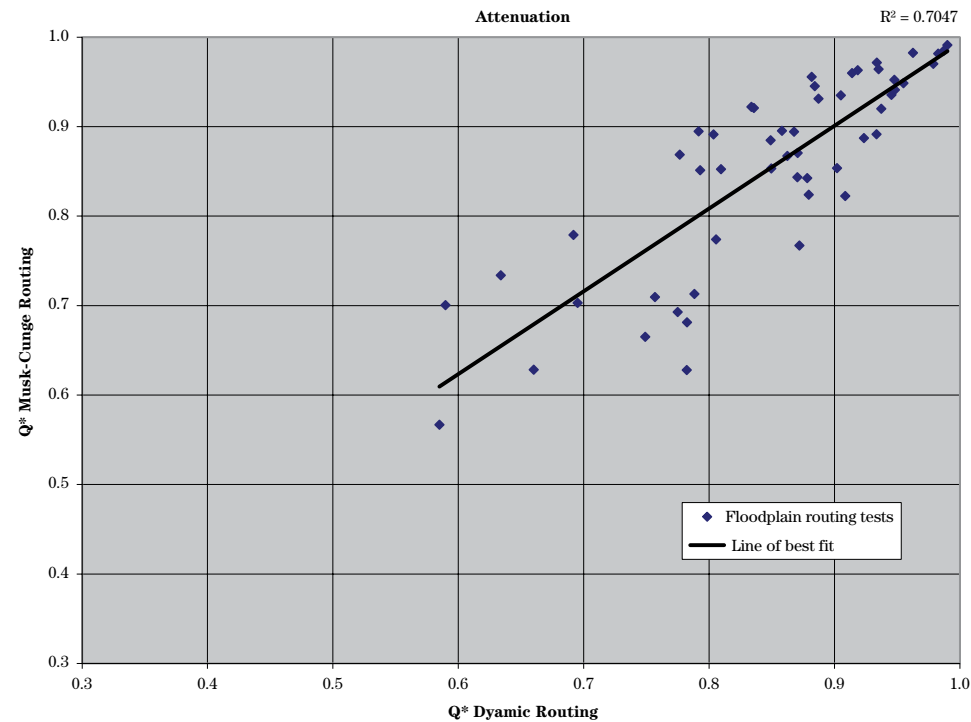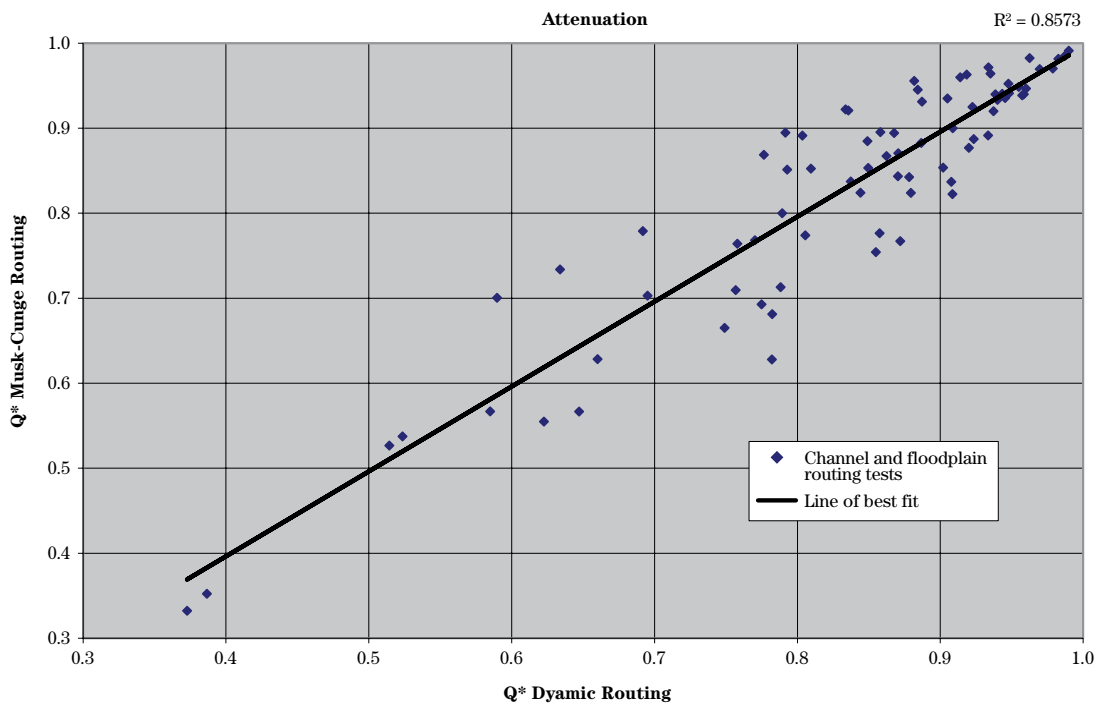**Figure 17A–2**   Comparison of peak discharges for 52 floodplain tests

Figure 17A–3   Comparison of peak discharges for 77 channel and floodplain tests

Appendix B   Limits of the Muskingum-Cunge Procedure

Based on Muskingum-Cunge routing coefficients computed from cross section, reach, and inflow hydrograph, the routing of the hydrograph may be inaccurate. Based on 140 routing tests, conditions have been identified when this may happen. A warning in WinTR–20 will alert the user when this may occur.

Figure 17B–1 is a plot where the outflow hydrograph is not accurate. The peak outflow discharge is significantly less than the peak inflow discharge but occurs at the same time. The WinTR–20 warning instructs the user to plot the inflow and outflow hydrographs for the reach. If this kind of plot results, two actions are recommended. One is to rerun the routings using an unsteady flow model such as HEC–RAS. The other is to convert the channel reach into a structure reach and rerun WinTR–20.

To convert the channel reach to a structure reach, revise the channel cross section table to a structure table by keeping the elevation and discharge columns of the channel cross section and converting the channel cross section end-area to storage. For English units, convert the column of area in square feet to acre-feet by multiplying by the reach length and dividing by 43,560. Use this table of elevation, discharge and storage to route the reach using the storage-indication method. For this example, converting the channel reach to a structure reach produces the plot in figure 17B–2.

This warning limit occurs mostly on long flat reaches. In this case, a structure routing provides a better answer than the Muskingum-Cunge routing.

The warning limit is triggered by the routing coefficients C_g and D_g. C_g is named the Courant number and D_g is named the grid Reynolds number. The word "grid" is included because the value is dependent upon the computational grid routing distance step. These coefficients are nondimensional and are defined as:

$$C_g = c_r \left(3,600\right)\frac{\Delta t}{\Delta x} \qquad \text{(eq. 17B–1)}$$

$$D_g = \frac{Q_{pi}}{T_r S_r c_r \Delta x} \qquad \text{(eq. 17B–2)}$$

In figure 17B–3, the 140 test routings were divided into good, fair and poor routing results. Based on figure 17B–3, the curve with the diamond shaped points marks the warning threshold. Values of C_g and D_g that fall above the curve cause the warning to be displayed. The relation of C_g and D_g coefficients that triggers the warning is:

$$D_g > e^{2.3 C_g} \qquad \text{(eq. 17B–3)}$$

For example, if $C_g = 1.0$, then if D_g is greater than $e^{2.3}$ or 9.97, the following warning is displayed:

Warning—Channel routing parameters Courant number and Grid Reynolds Number are beyond accuracy limits. View reach US/DS hydrograph plots for reasonableness. Storage or unsteady routing recommended for REACH R–29 and STORM 10–yr

The warning includes the reach identification and storm identification where the situation occurs. Note that not all routings above the curve are poor. In fact, there are several fair routings and one good routing above the curve. That is why it is recommended to plot the inflow and outflow hydrographs to determine if the routing should be revised. Combination of values of C_g and D_g should not fall below the line representing the lower limit of C_g and D_g in figure 17B–3 . For more information on the values of C_g and D_g and their significance, see Ponce (1981, 1983).

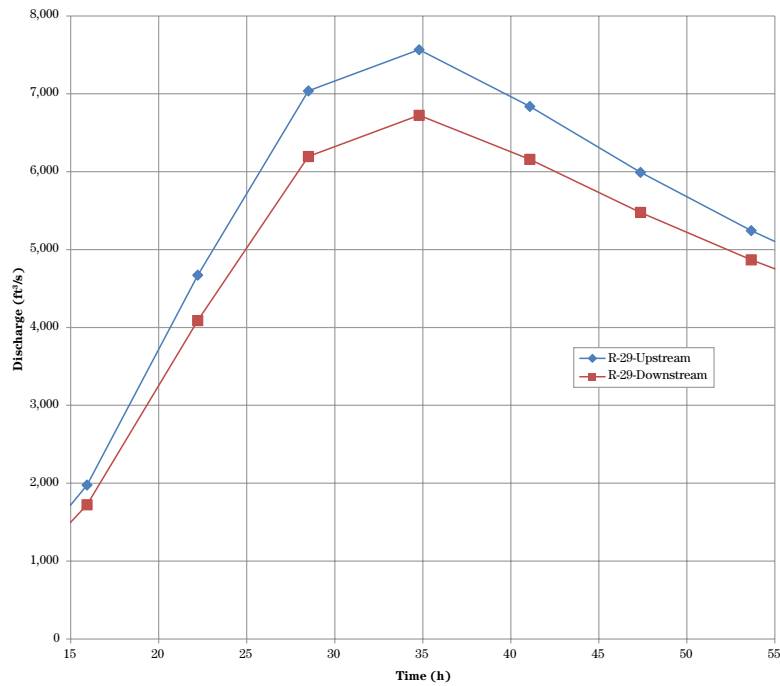**Figure 17B–1** Condition where Muskingum-Cunge warning limit is exceeded

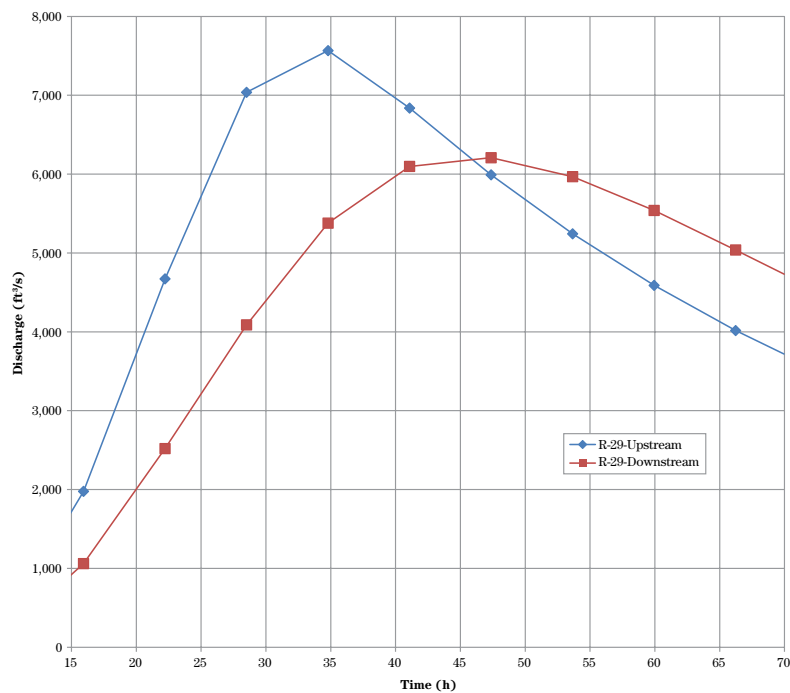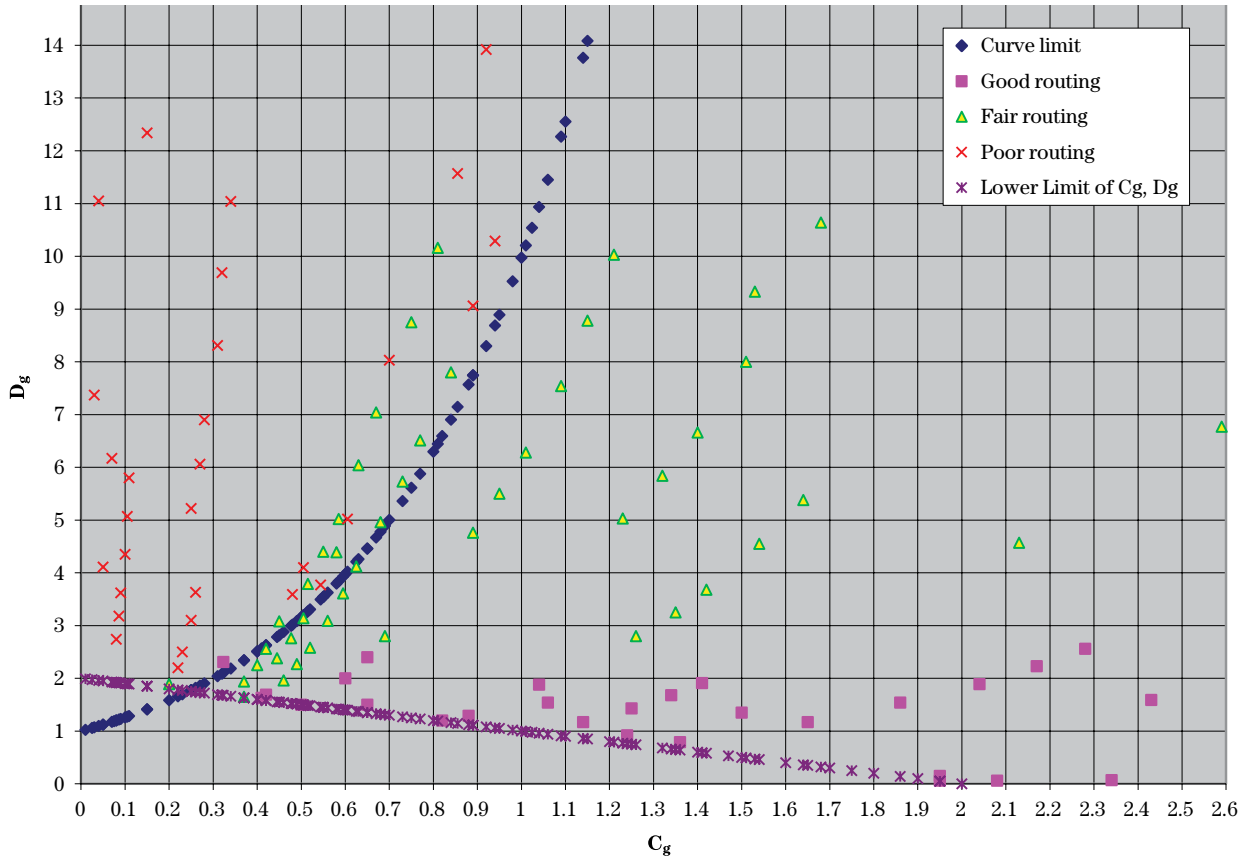**Figure 17B–2** Hydrograph plots when routing the reach as a structure

Figure 17B–3   Plot of C_g and D_g coefficients and the warning limit threshold

Appendix C — Determination of the Routing Distance Steps in the Muskingum-Cunge Procedure

Background and introduction

There is a long history of use of hydrologic routing methods in the NRCS technical tools. In the 1950s Victor Mockus derived the Convex routing method (SCS, 1965). In the late 1970s, Theurer and Comer derived the Modified Att-Kin Method (Comer et al. 1982). These models have particular characteristics that reveal limitations to the accuracy of routing results.

The Convex method routes a reach in a series of routing distance steps where the step length is based on the time interval. The limitation is that if the reach is routed at two different time intervals, there are a different number of steps, and the results at the end of the reach are different. In other words, the Convex routing is sensitive to the time interval used to route the hydrograph. This problem leads to the question, "What is the proper time interval to use to route the hydrograph?" There is no good technical answer to this question.

In an attempt to overcome these problems, the Modified Att-Kin method replaced the Convex method in the TR–20 version of 1983. The modified Att-Kin method routes the reach in a single step. The advantage over the Convex method is that if the time interval is changed, the results are still very close at the end of the reach. In other words, the Modified Att-Kin method is not sensitive to time interval changes. It was discovered sometime later that if the reach was divided into two or more steps, the routings at the end of the reach are different. This problem leads to the question What is the proper routing distance step interval to use to route the hydrograph? There is no good technical answer to this question.

The Muskingum-Cunge method overcomes both of these problems and is not sensitive to time interval or routing distance step. Ponce (1981, 1983) showed that results of the Muskingum-Cunge are consistent with respect to changes in time interval and routing distance step. This appendix explains the procedure used to estimate the routing distance step in WinTR–20.

Calculation of routing distance step

Ponce (1981, 1983) derived an equation to estimate the routing distance step for the Muskingum-Cunge method. The equation is :

$$\Delta x = 0.5 \left(c_r \Delta t + \frac{Q_{pi}}{T_r \left(S_r \right) c_r} \right) \qquad \text{(eq. 17C–1)}$$

The formulation of the Muskingum-Cunge method in WinTR–20 uses the peak of the inflow hydrograph as the reference discharge. The multiplier 0.5 in the equation is based on a series of tests by Ponce (1981, 1983). The equation has two distinct parts. The first term, $c_r \Delta t$, is easily visualized as the distance traveled by the flood wave in one time interval. The second term, $Q_{pi}/(T_r\ S_r\ c_r)$, is more difficult to visualize but simplifies into a depth of water divided by the slope.

If questions arise concerning the Muskingum-Cunge routing of particular reaches in WinTR–20, complete information for each reach may be directed to the Debug file. This information includes values to compute the routing distance step, values used to compute the routing coefficients, and step-by-step routing results for the reach if it is divided into a number of routing distance steps.

Appendix D   List of Variables

Variable	Definition	Equation no.
∂	Partial derivative operator	17–12, 17–13, 17–14, 17–15
ΔS	Change in volume of storage during a time interval, ft^3, in, ft^3/s-h, ft^3/s-days, acre-feet	17–1
Δt	Time interval $t_2 - t_1$, seconds, hours, days, or minutes	17–1, 17–2, 17–3, 17–5, 17–8, 17–9, 17–10, 17–11, 17–17, 17–21, 17–32, 17–33, 17B–1, 17C–1
Δx	Routing distance step, ft	17–17, 17–18, 17–19, 17–20, 17–21, 17–22, 17–32, 17–34, 17B–1, 17B–2, 17C–1
μ	Theoretical diffusivity, ft^2/s	17–16
μ_n	Numerical diffusivity, ft^2/s	17–18
A	End-area for discharge Q, ft^2	17–4, 17–23, 17–24, 17–25, 17—27, 17–28
A_r	Cross section end-area at the reference discharge, ft^2	17–31
c	Flood wave celerity, ft/s	17–17, 17–18, 17–19, 17–20, 17–21, 17–22, 17–24, 17–25, 17–26
C_0	Dimensionless coefficient	17–8, 17–9, 17–10, 17–11
C_1	Dimensionless coefficient	17–7, 17–8
C_2	Dimensionless coefficient	17–7, 17–9
C_3	Dimensionless coefficient	17–7, 17–10
C_g	Courant number, the ratio of the physical wave celerity to the grid celerity $\Delta x/\Delta t$, Dimensionless	17–32, 17–33, 17–36, 17B–1, 17B–3
c_r	Reference wave celerity (corresponding to peak of inflow hydrograph), ft/s	17–31, 17–32, 17–34, 17B–1, 17B–2, 17C–1
d	Depth of flow, ft	17–12, 17–13, 17–14, 17–15
d	Derivative operator	17–24, 17–25
D_g	Grid Reynolds number, a mathematical criterion which distinguishes laminar from turbulent flow, dimensionless	17–34, 17–35, 17–36, 17B–2, 17B–3
e	Base of natural logarithms, or approximately 2.71828.	17–36, 17B–3
g	Acceleration of gravity, ft/s^2	17–13
$\overline{I}$	Average rate of inflow during a time interval, ft^3/s	17–1
I	Inflow discharge, ft^3/s	17–6
I	Flow number	17–30a, 17–30b
I_1	Inflow rate at t_1, ft^3/s	17–2, 17–3, 17–5, 17–7, 17–17
I_2	Inflow rate at t_2, ft^3/s	17–2, 17–3, 17–5, 17–7, 17–17
K	Storage constant, s	17–6, 17–8, 17–9, 17–10, 17–11, 17–22, 17–33
L	Reach length, ft	17–4
m	Exponent	17–23, 17–25, 17–26, 17–28, 17–29a, 17–31
$m(I)$	m at flow number I	17–30a

Variable	Definition	Equation no.
n	Manning's roughness coefficient, dimensionless	17–27
O	Outflow discharge, ft^3/s	17–6
O_1	Outflow rate at t_1, ft^3/s	17–2, 17–3, 17–5, 17–7, 17–17
O_2	Outflow rate at t_2, ft^3/s	17–2, 17–3, 17–5, 17–7, 17–17
$\bar{O}$	Average rate of outflow during a time interval, ft^3/s	17–1
p	Wetted perimeter, ft	17–27
Q	Flow rate or discharge, ft^3/s	17–4, 17–14, 17–16, 17–19, 17–20, 17–21, 17–23, 17–24, 17–25, 17–27, 17–28, 17–29b, 17–30b
Q*	Discharge ratio calculated from peak discharges resulting from Muskingum-Cunge routings or from the dynamic wave model (FLDWAV), dimensionless	17A–1
Q(I)	Discharge at flow number I	17–29b, 17–30a, 17–30b
Q_b	Baseflow, ft^3/s	17A-1
Q_{pi}	Peak discharge of the inflow hydrograph, reference discharge, ft^3/s	17–31, 17–34, 17A-1, 17B–2, 17C–1
Q_{po}	Peak discharge of the outflow hydrograph, ft^3/s	17A–1
S	Reach storage, ft^3	17–6
S_r	Reference energy grade slope (slope corresponding to peak of inflow hydrograph), ft/ft	17–34, 17B–2, 17C–1
S(2,3)	Log-log slope of discharge-end-area curve between points 2 and 3, dimensionless	17–29a, 17–30a
S(I–1,I)	Log-log slope of discharge-end-area curve between points I–1 and I	17–30a
S_1	Storage volume at t_1, ft^3/s-h	17–2, 17–3, 17–5
S_2	Storage volume at t_2, ft^3/s-h	17–2, 17–3, 17–5
S_f	Friction slope, ft/ft	17–13, 17–15, 17–27
S_o	Slope of channel in longitudinal direction, ft/ft	17–13, 17–15, 17–16, 17–19, 17–20, 17–21
t	Time, s	17–12, 17–13, 17–14
t_1	Time at the beginning of a time interval Δt, h	17–1
t_2	Time at the end of a time interval Δt, h	17–1
T	Top width of cross section, ft	17–14, 17–16, 17–19, 17–20, 17–21
T_r	Reference top width (top width at peak of inflow hydrograph), ft	17–34, 17B–2, 17C–1
T_t	Reach travel time in hours: the time it takes a selected steady flow discharge to pass through the reach	17–4
V	Q/A, velocity of discharge or flow, ft/s	17–4, 17–12, 17–13, 17–26
X	Weighting factor, dimensionless	17–6, 17–8, 17–9, 17–10, 17–11, 17–17, 17–18, 17–19, 17–20, 17–35
x	Distance, ft	17–12, 17–13, 17–14, 17–15
x	Coefficient	17–23, 17–25, 17–28